Class_14 Oct 25 More Trig Identities

Monday, October 24, 2022 1:53 PM

Tonight's Class:

- **Test Warm-up**
- **Chapter 5 Test**
- **6.2 Identities to get Exact Values**
- **6.3 Proving Identities**

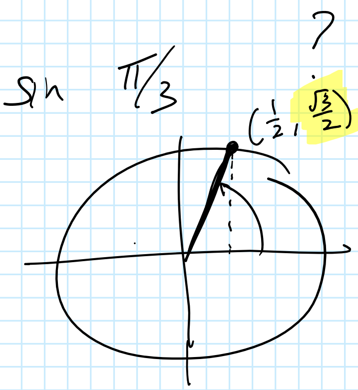
Please:

1. **Make sure your name is on your Chapter 5 Hand-in, hand it in.**
2. **Put away your calculator and all other materials.**
3. **On your test, write clearly and show all necessary steps.**
   When you are finished the non-calculator portion, bring it to me & I'll give you the rest of the test. You can use a calculator for the second part.
4. **While other people are still finishing, respect them by being quiet. You can leave the classroom if you wish, but be back in time for the rest of class.**

Ch 4

Exact value of $\sin \pi/3$

$= \boxed{\frac{\sqrt{3}}{2}}$



Pre-Calc 12 – Unit 2
Page 46

6.2 Sum, Difference, and Double-Angle Identities

Sum/Difference Identities

45° 150°

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ $\quad$ $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ $\quad$ $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Double Angle Identities

$2\tan\theta$

$1+\tan\alpha\tan\beta$

Double Angle Identities

$\sin(2\theta)=2\sin\theta\cos\theta$ $\quad\cos(2\theta)=\cos^2\theta-\sin^2\theta$ $\quad\tan(2\theta)=\dfrac{2\tan\theta}{1-\tan^2\theta}$

$\cos(2\theta)=2\cos^2\theta-1$

$\cos(2\theta)=1-2\sin^2\theta$

Examples

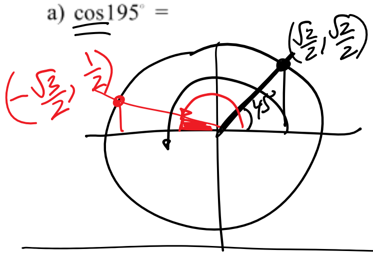
Find the exact value of the following expressions.

a) $\cos 195^\circ =$

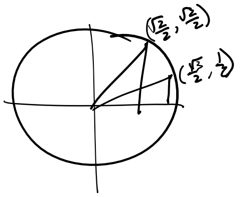
$\cos(45^\circ+150^\circ) = \cos 45^\circ \cos 150^\circ - \sin 45^\circ \sin 150^\circ$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$ OR $\frac{-\sqrt{6}-\sqrt{2}}{4}$



b) $\sin\left(\frac{5\pi}{12}\right)=$ can change $\frac{5\pi}{12}$ to degree: $\frac{5\pi}{12}\cdot\frac{180}{\pi} = 75^\circ$



$\sin(75^\circ) = \sin(30^\circ+45^\circ)$

$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

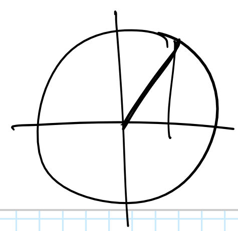
$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}$ OR $\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

c) $\underbrace{\sin 50^\circ \cos 10^\circ + \cos 50^\circ \sin 10^\circ}_{\sin(\alpha+\beta)} =$

$= \sin(50^\circ+10^\circ)$

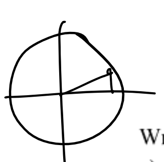
$= \sin(60^\circ) = \frac{\sqrt{3}}{2}$



<u>Using Identities</u> <u>to Find Exact Values</u>

Use an identity to write the given angle as the sum or difference of 2 angles.

The two angles you choose have to be either special triangle angles, or angles that have a special triangle angle as its reference angle.



Write each expression in a simpler form, using identities. Give exact value, if possible.

a) $2\sin 15^\circ \cos 15^\circ =$ $\sin(2\times 15^\circ)$

b) $\cos^2(\pi/8)-\sin^2(\pi/8)=$ $\cos\left(\frac{2}{1}\times\frac{\pi}{8}\right)$

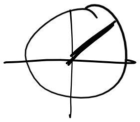
Write each expression in a simpler form, using identities. Give exact value, if possible.

a) $2\sin 15^\circ \cos 15^\circ =$ $\sin(2 \times 15^\circ)$

$= \sin(30^\circ)$

$= \frac{1}{2}$

b) $\cos^2(\pi/8) - \sin^2(\pi/8) =$ $\cos\left(\frac{2}{1} \times \frac{\pi}{8}\right)$



$= \cos\left(\frac{2\pi}{8}\right)$

$= \cos\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2}$

c) $1 - 2\sin^2(8) =$ $\cos(2 \times 8)$

$= \cos(16)$

d) $10\cos^2(x) - 5 =$

$5(2\cos^2(x) - 1)$

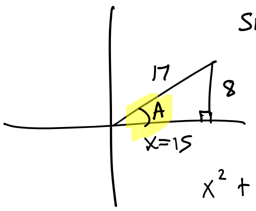
$= 5(\cos 2x)$

$= 5\cos 2x$

Given that $\sin A = \frac{8}{17}$, where A is a Q1 angle; and $\cos B = \frac{24}{25}$, where B is a Q4 angle.

a) Draw two coordinate systems. Sketch a reference triangle for angle A on one of the systems, and one for angle B on the other.

$\sin A = \frac{8}{17}$ ← opp, ← hyp

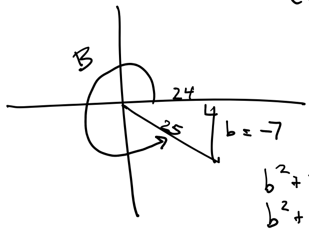


$x^2 + 8^2 = 17^2$

$x^2 + 64 = 289$

$x^2 = 225, \quad x = \pm 15$

$\cos B = \frac{24}{25}$ ← adj, ← hyp



$b^2 + 24^2 = 25^2$

$b^2 + 576 = 625$

$b^2 = 49$

$b = \pm 7$

b) Use identities to find the exact value of:

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$= \left(\frac{8}{17}\right)\left(\frac{24}{25}\right) - \left(\frac{15}{17}\right)\left(\frac{-7}{25}\right)$

$= \frac{192}{425} - \left(\frac{-105}{425}\right)$

$= \frac{192}{425} + \frac{105}{425} = \boxed{\frac{297}{425}}$

$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$

$= \frac{\frac{2}{1}\left(\frac{8}{15}\right)}{1 - \left(\frac{8}{15}\right)^2}$

$= \frac{\frac{16}{15}}{1 - \frac{64}{225}}$

$\frac{\frac{16}{15}}{\frac{225}{225} - \frac{64}{225}}$

$= \frac{\frac{16}{15}}{\frac{161}{225}}$

$= \frac{16}{\cancel{15}} \cdot \frac{\cancel{225}^{15}}{161} = \boxed{\frac{240}{161}}$

$\sin 2B = 2\sin B\cos B$

$= \frac{2}{1}\left(\frac{-7}{25}\right)\left(\frac{24}{25}\right)$

$= \boxed{\frac{-336}{625}}$

$\cos 2A$ = whichever version you want!

$= \cos^2 A - \sin^2 A$

$= \left(\frac{15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$

$= \frac{225}{289} - \frac{64}{289}$

$= \boxed{\frac{161}{289}}$

OR $2\cos^2 A - 1$

$= 2\left(\frac{15}{17}\right)^2 - 1$

$= \frac{2}{1}\left(\frac{225}{289}\right) - \frac{289}{289}$

$= \frac{450}{289} - \frac{289}{289}$

$= \frac{161}{289}$

(6.2) TB p 306: 1ade, 2ac, 4ace, 5, 8ace, 10, 11, 16, 20acd

Identities are useful for calculus:

[http://people.wallawalla.edu/~thomth/H200/21TrigIdentities.pdf]

6.3 Proving Identities

When we prove identities:

Show the process

- Step by step, use algebra and/or Basic Identities to change the way either the left- hand side (LHS) or the right-hand side (RHS) looks.
- Think of the " = " sign separating the LHS and RHS as a barrier. Don't take terms from one side of the equals sign to the other.
- When the LHS and the RHS look exactly the same, the identity is proven.

Strategies for Proofs

- Write each step directly below the previous one.
- Don't skip steps – aim to be as CLEAR as possible.
- See if there's any factoring you can do, especially GCF or difference of squares.
- Don't cancel anything, unless you have identical factors on the top and bottom of an expression.
- If rational expressions are added /subtracted together, get a common denominator so you can combine the expressions and simplify.
- If possible, substitute known identities to simplify expressions.
- If the LHS and RHS look as below, where they are ***almost*** reciprocals of each other, multiply one side, top and bottom, by the conjugate of the binomial. Then use a Pythagorean identity to simplify further.

$$\frac{\cos y}{1-\sin y} = \frac{1+\sin y}{\cos y}$$

Try these strategies to prove the identities on the following pages.

1. $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

$= (\sin x + \cos x)(\sin x + \cos x)$

$= \sin^2 x + \sin x \cos x + \cos x \sin x + \cos^2 x$

$= 2\sin x \cos x + \sin^2 x + \cos^2 x$

$= 2\sin x \cos x + 1$

$= 1 + 2\sin x \cos x$

2. $\tan^2 x \sin^2 x - \tan^2 x = -\sin^2 x$

$= \tan^2 x(\sin^2 x - 1)$

$= \tan^2 x(-\cos^2 x)$

$= \left(\frac{\sin^2 x}{\cancel{\cos^2 x}}\right)\left(\frac{-\cancel{\cos^2 x}}{1}\right)$

$= -\sin^2 x$

we used an identity substitution here!

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

Re-arrange it:

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta - 1 = -\cos^2\theta$

3. $\sec^4 x = \tan^4 x + 2\tan^2 x + 1$

4. $(3 - 3\sin x)(3 + 3\sin x) = 9\cos^2 x$

Practice Questions

- **(6.1) TB p 296: 3, 4, 5a, 6b, 10, 14-16 **Don't have to identify NPVs**
- **(6.2) TB p 306: 1ade, 2ac, 4ace, 5, 8ce, 10, 11, 16, 20acd**
- **(6.3) TB p 314: 2, 3ac, 5, 7, 10c, 11a, 12a, 15b, 18**

Chapter 6 Hand-in - due Tuesday, November 1

Unit 2 Test (Chapters 4, 5, 6) Tuesday, November 1

My plan for that class is to talk briefly about section 7.1, then use the rest of the class for the Unit 2 Test. When finished the test, you will be free to leave.

Around 40 marks on this test, about 15 multiple-choice questions and the rest from written.

Know how to:

- Convert between degree and radian measure
- Graph angles in standard position
- Determine coterminal angles and reference angles
- Solve problems involving arc length
- Use trigonometric ratios with exact triangles
- Use the unit circle
- Solve trigonometric equations
- Graph the sine and cosine functions
- Perform transformations on sinusoidal functions
- Model real situations with sinusoidal functions
- Verify trigonometric identities
- Explore equations using Pythagorean identities
- Apply Sum and Difference identities to expressions
- Prove identities
- Use identities to help solve trigonometric equations

Things you can do to prepare

- **Worksheet: Unit 2 Review Questions**
- **Extra Practice 6.1, 6.2**
- **Extra Practice 6.3, 6.4**
- **Textbook practice questions from this unit**
- **Unit 2 review from the textbook (pages 326-329)**