Class_15 Oct 27 Trig Id for Equations

Sunday, October 16, 2022 4:09 PM

Tonight's Class

- **Chapter 5 Test return**
- **6.3 Proving Identities (continued)**
- **Solving Trig Equations Using Identities**
- **Unit 2 Test - Tuesday, November 1**

On your small whiteboard

- **write two things from Chapter 5 you plan to strengthen, before the Unit 2 Test.**

What can help us get more comfortable with math questions?

- **Do more of them**
- **Sleep on it**

1. $(\sin x + \cos x)^2 = 1 + 2\sin x\cos x$

$= (\sin x + \cos x)(\sin x + \cos x)$

$= \sin^2 x + \sin x\cos x + \cos x\sin x + \cos^2 x$

$= 2\sin x\cos x + \sin^2 x + \cos^2 x$

$= 2\sin x\cos x + 1$

$= 1 + 2\sin x\cos x$

2. $\tan^2 x\sin^2 x - \tan^2 x = -\sin^2 x$

$= \tan^2 x(\sin^2 x - 1)$

$= \tan^2 x(-\cos^2 x)$

$= \left(\frac{\sin^2 x}{\cancel{\cos^2 x}}\right)\left(\frac{-\cancel{\cos^2 x}}{1}\right)$

$= -\sin^2 x$

we used an identity substitution here!

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

re-arrange it:

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta - 1 = -\cos^2\theta$

3. $\sec^4 x = \tan^4 x + 2\tan^2 x + 1$

$(\tan^2 x + 1)(\tan^2 x + 1)$

$(\sec^2 x)(\sec^2 x)$

$\sec^4 x$

4. $(3 - 3\sin x)(3 + 3\sin x) = 9\cos^2 x$

$9 + \cancel{9\sin x} - \cancel{9\sin x} - 9\sin^2 x$

$9 - 9\sin^2 x$

$9(1 - \sin^2 x)$

$9\cos^2 x$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\cos^2\theta = 1 - \sin^2\theta$

TB, page 311

Your Turn

Prove that $\frac{\sin 2x}{\cos 2x + 1} = \tan x$ is an identity for all permissible values of x.

$$\frac{2\sin x\cos x}{2\cos^2 x \cancel{-1} + \cancel{1}}$$

$$\frac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x} \qquad \Bigg| \qquad \frac{\sin x}{\cos x}$$

$$\frac{\sin x}{\cos x}$$

Remember how we get common denominators when we add or subtract fractions:

$$\frac{4}{4}\cdot\frac{2}{5} + \frac{3}{4}\cdot\frac{5}{5}$$

$$= \frac{8}{20} + \frac{15}{20} = \frac{8+15}{20}$$

$$= \frac{23}{20}$$

We use the same method to simplify identities that are rational expressions.
This is often helpful when we try to prove an identity.

5. $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} \qquad = \qquad 2\csc^2 x$

$$\frac{(1-\cos x)}{(1-\cos x)}\cdot\frac{1}{1+\cos x} + \frac{1}{1-\cos x}\cdot\frac{(1+\cos x)}{(1+\cos x)}$$

$$\frac{1-\cos x}{(1-\cos x)(1+\cos x)} + \frac{1+\cos x}{(1-\cos x)(1+\cos x)}$$

$$\frac{1-\cancel{\cos x}+1+\cancel{\cos x}}{(1-\cos x)(1+\cos x)}$$

$$\frac{2}{(1-\cos x)(1+\cos x)}$$

$$\frac{2}{1+\cancel{\cos x}-\cancel{\cos x}-\cos^2 x}$$

$$\frac{2}{1-\cos^2 x}$$

$$\frac{2}{\sin^2 x}$$

$$2\cdot\frac{1}{\sin^2 x}$$

$$2\csc^2 x \qquad \Bigg| \qquad 2\csc^2 x$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

What is a conjugate? How does multiplying by it help?

- **The conjugate of a binomial looks exactly like the binomial, except that the sign between the two terms is OPPOSITE from what it is in the original binomial.**

example: $2x - 7$ and $2x + 7$

- **When we multiply an expression, top & bottom, by the conjugate of the binomial that is in the numerator or denominator, we are really multiplying by 1.**

- **The resulting expression can be written in a different form, by using a Pythagorean Identity.**

5. $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

Done above

conjugate

(multiply these out)

6. $\frac{\cos y}{1-\sin y} \cdot \frac{(1+\sin y)}{(1+\sin y)} = \frac{1+\sin y}{\cos y}$

$\frac{\cos y\,(1+\sin y)}{1+\cancel{\sin y}-\cancel{\sin y}-\sin^2 y}$

$\frac{\cos y\,(1+\sin y)}{1-\sin^2 y}$

$\frac{\cancel{\cos y}\,(1+\sin y)}{\cos^{\cancel{2}} y}$

$\frac{1+\sin y}{\cos y}$

7. $2\sec x = \frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$

8. $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

$\frac{\sin x}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x}$

$\frac{\sin x\,(1-\cos x)}{(1+\cos x)(1-\cos x)}$

$\frac{\sin x\,(1-\cos x)}{1-\cancel{\cos x}+\cancel{\cos x}-\cos^2 x}$

$\frac{\sin x\,(1-\cos x)}{1-\cos^2 x}$

$\frac{\cancel{\sin x}\,(1-\cos x)}{\sin^{\cancel{2}} x}$

$1-\cos x$

$\frac{\ldots}{\sin^2 x}$

$\frac{1-\cos x}{\sin x}$

7. $2\sec x \quad = \quad \frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$

$\frac{\cos x}{\cos x} \cdot \frac{(\cos x)}{(1+\sin x)} + \frac{(1+\sin x)}{\cos x} \cdot \frac{(1+\sin x)}{(1+\sin x)}$

$\frac{\cos^2 x}{\cos x(1+\sin x)} + \frac{1+\sin x+\sin x+\sin^2 x}{\cos x\,(1+\sin x)}$

$\frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x\,(1+\sin x)}$

$\frac{1 + 1 + 2\sin x}{\cos x\,(1+\sin x)}$

$\frac{2+2\sin x}{\cos x\,(1+\sin x)}$

$\frac{2\cancel{(1+\sin x)}}{\cos x\,\cancel{(1+\sin x)}}$

$\frac{2}{\cos x}$

$2 \cdot \frac{1}{\cos x}$

$2\sec x$

$\frac{8}{20} + \frac{15}{20}$

$\frac{8+15}{20}$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$2x+12$

$= 2(x+6)$

[Practice](#)

(6.3) TB p 314: 2, 3ac, 5, 7, 10c, 11a, 12a, 15b, 18

6.4 Solving Trigonometric Equations Using Identities

Some trigonometric equations cannot be solved until they are re-written in a different form, using trigonometric identities.

Example

Algebraically solve this equation, giving the ***general solution***, in radian measure.

$$2\sin x = 9 - 4\csc x$$

Use an identity to change how the equation looks.

$$\sin x \left[2\sin x = 9 - 4\left(\frac{1}{\sin x}\right) \right]$$

(This means $\sin x \neq 0$, so some x-values would not be permitted)

Eliminate the fractions:

$$2\sin^2 x = 9\sin x - 4$$

$-9\sin x + 4 \quad -9\sin x \quad +4$

$$2\sin^2 x - 9\sin x + 4 = 0$$

AC: 8
mult to 8
add to −9 } −8, −1

Factor

$$2\sin^2 x - 8\sin x - 1\sin x + 4 = 0$$

$$2\sin x(\sin x - 4) - 1(\sin x - 4) = 0$$

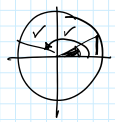
$$(\sin x - 4)(2\sin x - 1) = 0$$

$$(2\sin x - 1)(\sin x - 4) = 0$$

$$2\sin x - 1 = 0$$

$$\frac{2\sin x}{2} = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$



$\frac{\pi}{6}$ and $\frac{5\pi}{6}$

$$\sin x - 4 = 0$$

$$\sin x = \frac{4}{1}$$ ← opp, ← hyp

no solution

$$\boxed{\left.\begin{array}{l} \frac{\pi}{6} + 2\pi n \\ \frac{5\pi}{6} + 2\pi n \end{array}\right\} \; n \in I}$$

Example

Algebraically solve this equation

$$\cos 2x + \cos x = -1, \text{ for } 0^\circ \le x < 360^\circ$$

$$2\cos^2 x - 1 + \cos x = -1$$

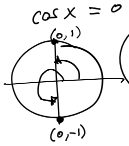
$+1 \quad +1$

$$2\cos^2 x \cancel{-1} + \cos x \cancel{+1} = 0$$

$$2\cos^2 x + \cos x = 0$$

$$\cos x(2\cos x + 1) = 0$$

$$\cos x = 0$$



(0, 1)
(0, −1)

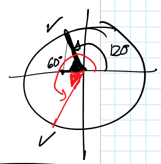
$x = 90^\circ$
270°

$$2\cos x + 1 = 0$$

$-1 \quad -1$

$$\frac{2\cos x}{2} = \frac{-1}{2}$$

$$\cos x = -\frac{1}{2}$$



$\boxed{120^\circ \text{ and } 240^\circ}$

Try

Algebraically solve this equation

$$2\cos x + 1 - \sin^2 x = 3 \text{ , for } 0 \le x < 2\pi$$

$2\cos x + \cos^2 x = 3$

$\cos^2 x + 2\cos x - 3 = 0$

AC = 1 × -3 = -3

add to 2

3, -1

$\cos^2 x + 3\cos x - 1\cos x - 3 = 0$

$\cos x(\cos x + 3) - 1(\cos x + 3) = 0$

$(\cos x + 3)(\cos x - 1) = 0$

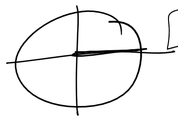
$\cos x + 3 = 0$

$\cos x = -3$

no solution

$\cos x - 1 = 0$

$\cos x = 1$



$\boxed{x = 0}$

- (6.4) TB p 320: 1bc, 2bc, 3abc, 4, 5, 10, 14

Work on these:
- Chapter 6 hand-in
- Unit 2 Review (posted online)

Chapter 6 Hand-in - due Tuesday, November 1

Unit 2 Test (Chapters 4, 5, 6) Tuesday, November 1

My plan for that class is to briefly talk about 7.1, then use the rest of the class for the Unit 2 Test. When finished the test, you will be free to leave.

Around 40 marks on this test, about 15-20 multiple-choice questions and the rest from written.

Know how to:

C

- Convert between degree and radian measure
- Graph angles in standard position
- Determine coterminal angles and reference angles
- Solve problems involving arc length
- Use trigonometric ratios with exact triangles
- Use the unit circle
- Solve trigonometric equations
- Graph the sine and cosine functions
- Perform transformations on sinusoidal functions
- Model real situations with sinusoidal functions
- Verify trigonometric identities
- Explore equations using Pythagorean identities
- Apply Sum and Difference identities to expressions
- Prove identities
- Use identities to help solve trigonometric equations

Things you can do to prepare

- **Worksheet: Unit 2 Review Questions**
- **Extra Practice 6.1, 6.2**
- **Extra Practice 6.3, 6.4**
- **Textbook practice questions from this unit**
- **Unit 2 review from the textbook (pages 326-329)**