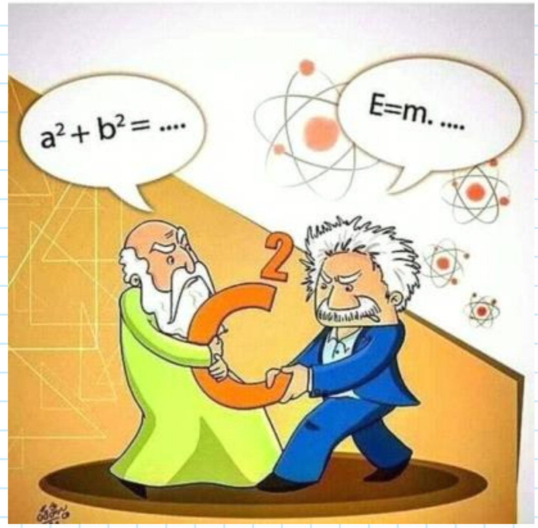
Class_16 June 1 Solving Exponential Functions & Intro to Logarithms



Plan For Today:

1. Question about anything from last class? 7.1-7.2
2. Finish Chapter 7: Exponential Functions
   - ✓ 7.1: Characteristics of Exponential Functions
   - ✓ 7.2: Transformations of Exponential Functions
   - ❖ **7.3: Solving Exponential Equations**
3. Work on practice questions from Textbook

Page 364: #1, 2, 3ac, 4, 5ac, 7aceg, 9-13

4. Start Chapter 8: Logarithmic Functions
   - ❖ **8.1: Understanding Logarithms**
   - ❖ 8.2: Transformations of Logarithmic Functions
   - ❖ 8.3: Laws of Logarithms
   - ❖ 8.4: Logarithmic & Exponential Functions
5. Work on practice questions from Textbook

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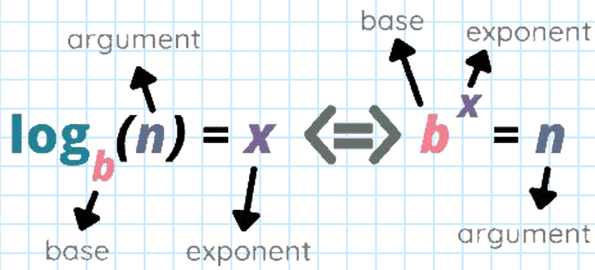

Plan Going Forward:

1. Finish working through textbook question from 7.3 & 8.1 and finish working on the Ch. 7 Assignment.

- ❖ CHAPTER 7 ASSIGNMENT DUE MONDAY, JUNE 6TH
- ❖ TEST 5 ON 6.4-8.2 ON MONDAY, JUNE 6TH

2. You will go over 8.1-8.2 Logs and graphing transformations of log functions tomorrow.

Please let us know if you have any questions or concerns about your progress in this course. The notes from today will be posted at egolfmath.weebly.com after class.

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Review

algebra2_e
xplog_gra...

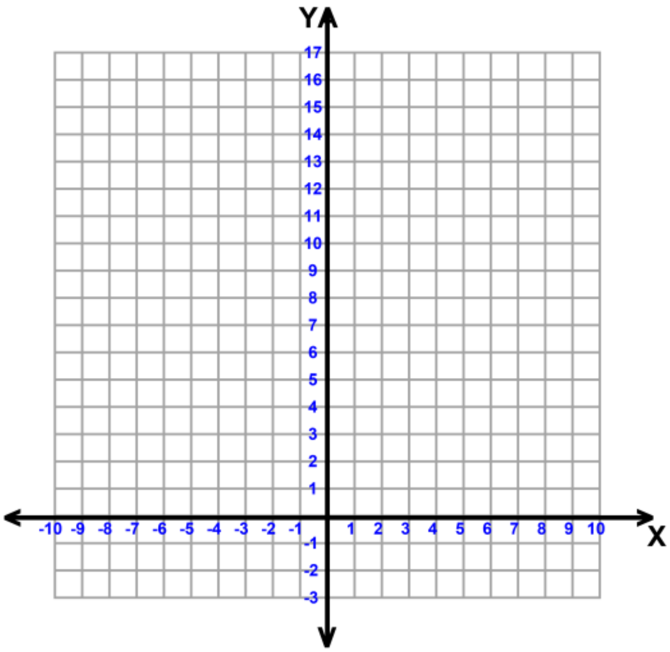
Name : ______________________________ Score : ____________

Teacher : ____________________________ Date : ____________

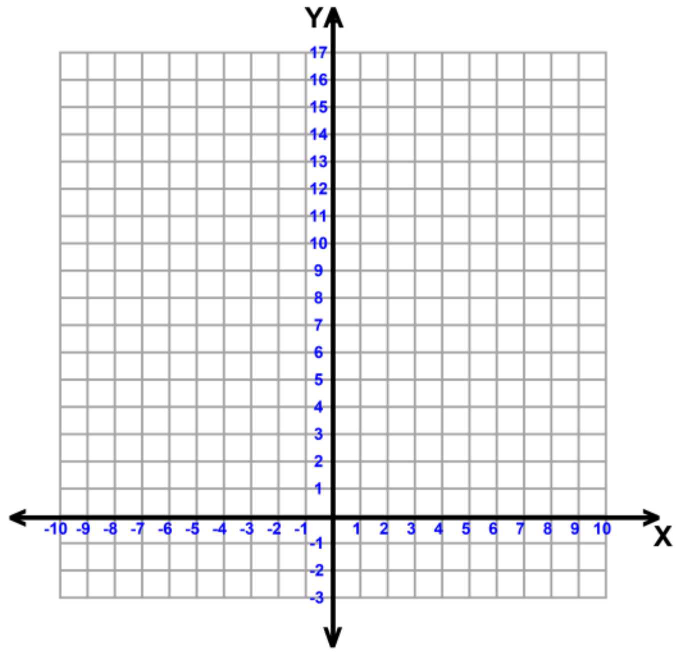
Graphing Exponential Functions

Sketch the graph of each function.

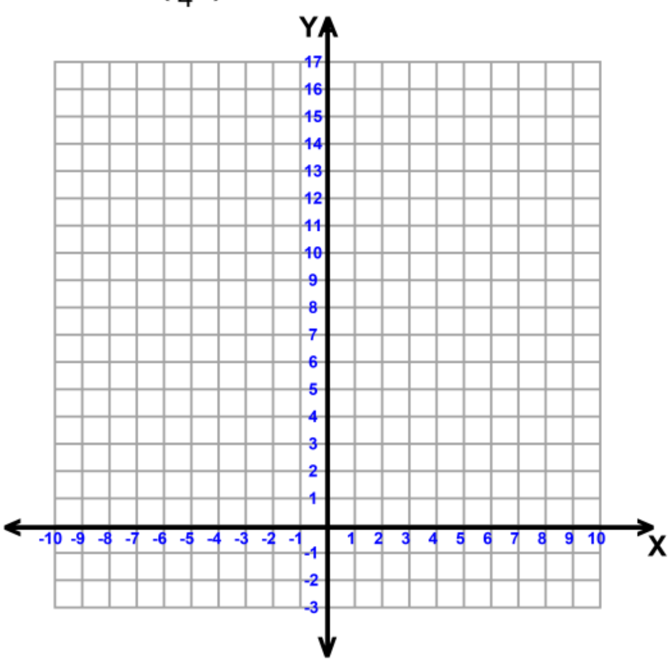
1) $y = 3 \cdot \left(\frac{1}{2}\right)^{x+3} - 3$



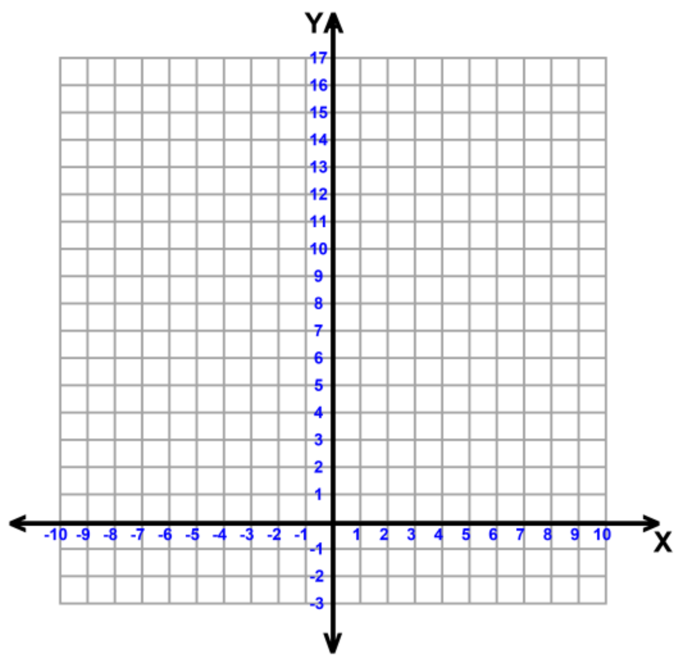
2) $y = 4 \cdot 2^{x+3} - 3$



3) $y = 3 \cdot \left(\frac{1}{4}\right)^{x}$



4) $y = 4 \cdot 3^{x}$



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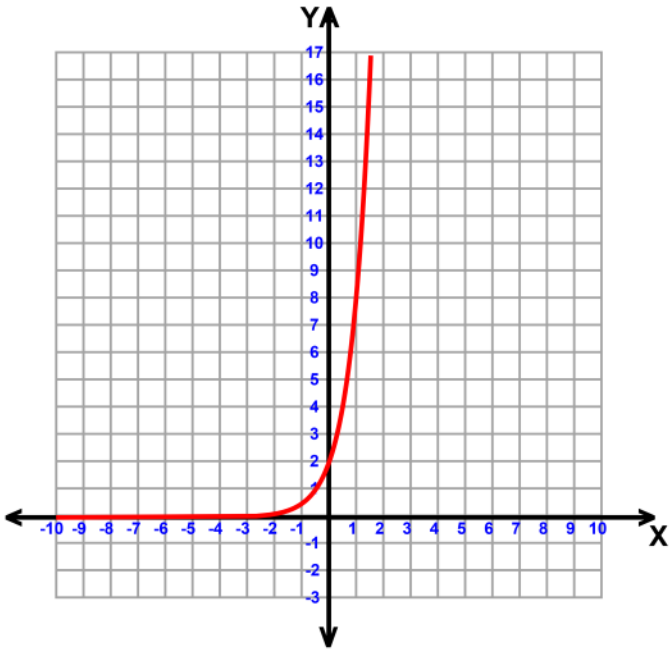
Graphing Exponential Functions

Write an equation for each graph.

5)



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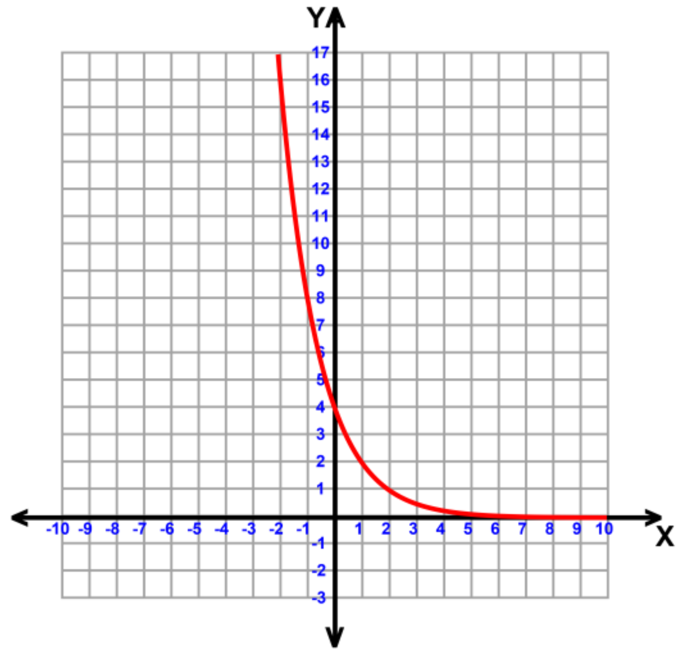

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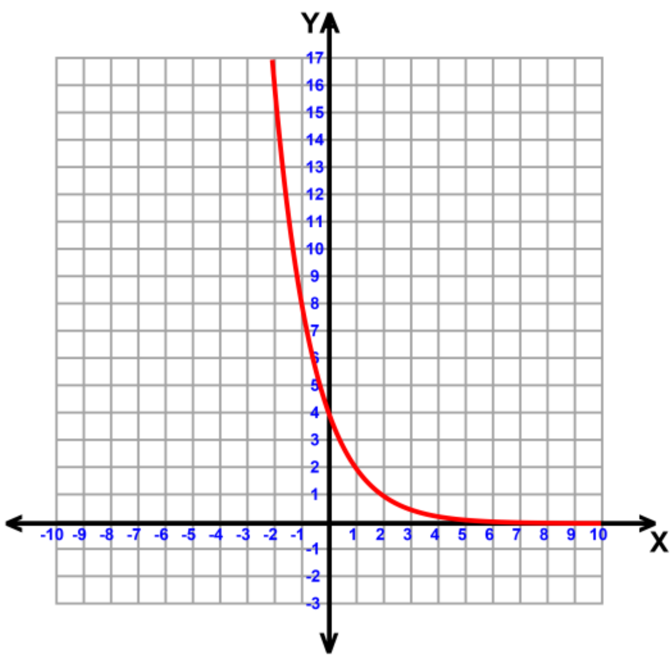

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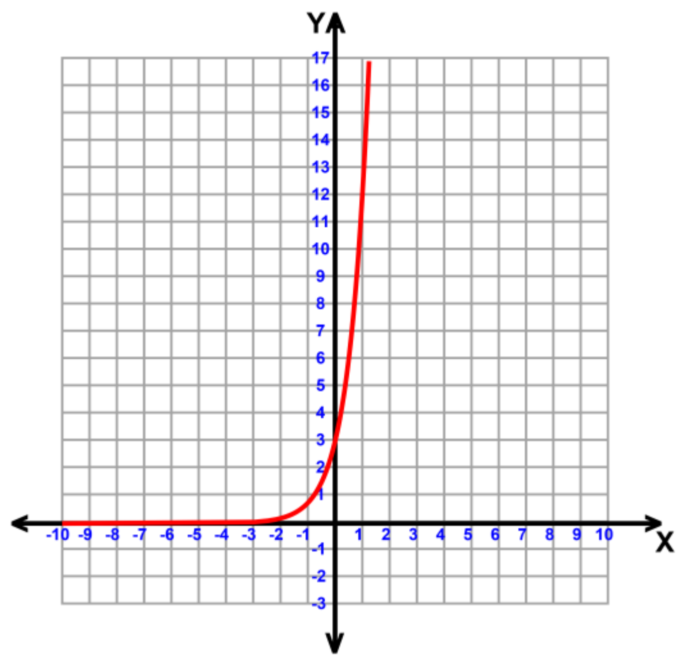

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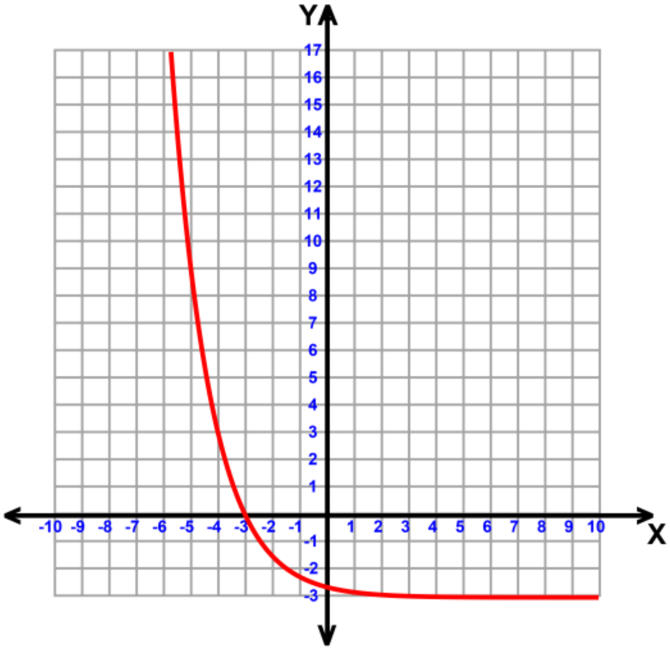
Teacher : ____________________________ Date : ____________

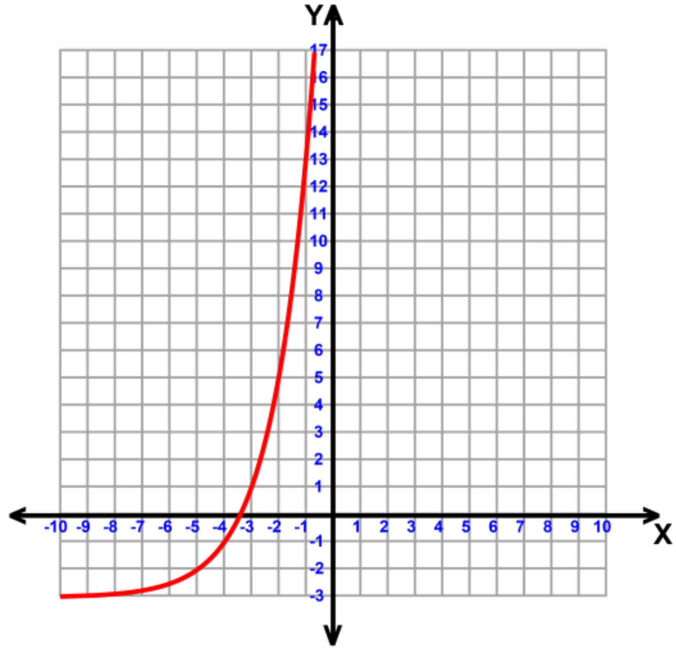
Graphing Exponential Functions

Sketch the graph of each function.

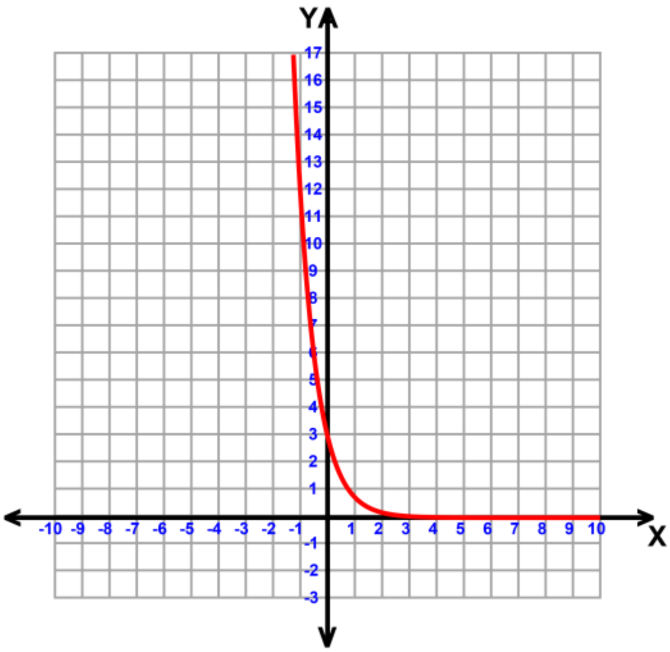
1) $y = 3 \cdot \left(\frac{1}{2}\right)^{x+3} - 3$



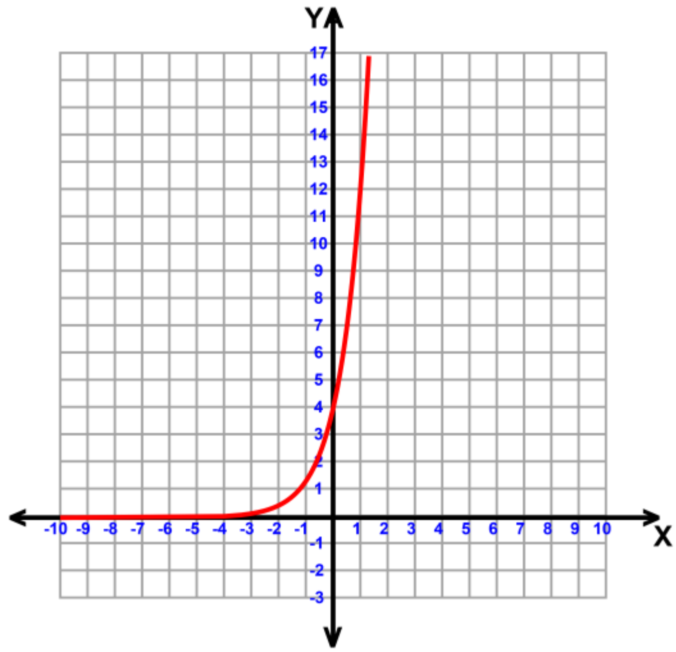
2) $y = 4 \cdot 2^{x+3} - 3$



3) $y = 3 \cdot \left(\frac{1}{4}\right)^{x}$



4) $y = 4 \cdot 3^{x}$



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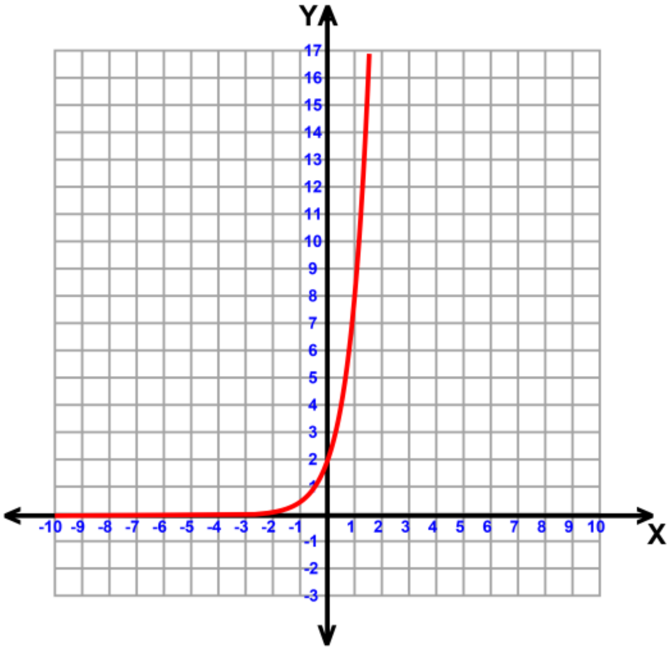
Teacher : ______________________ Date : __________

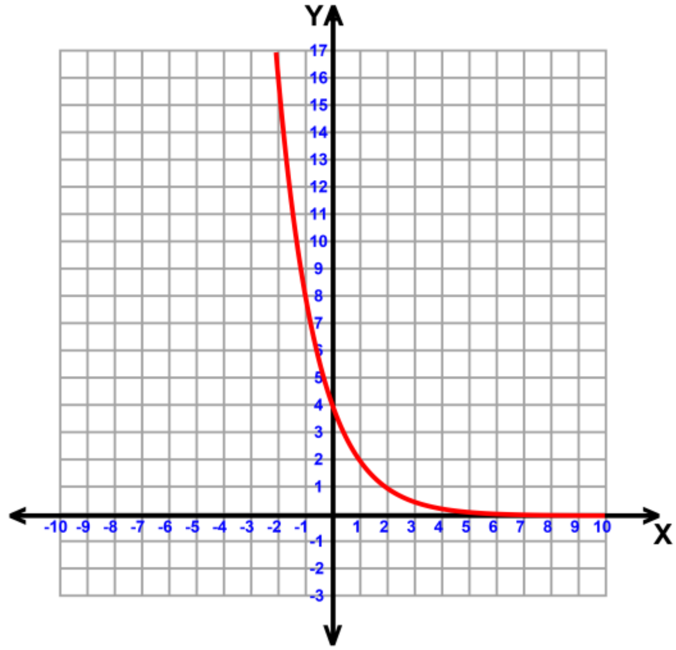
Graphing Exponential Functions

Write an equation for each graph.

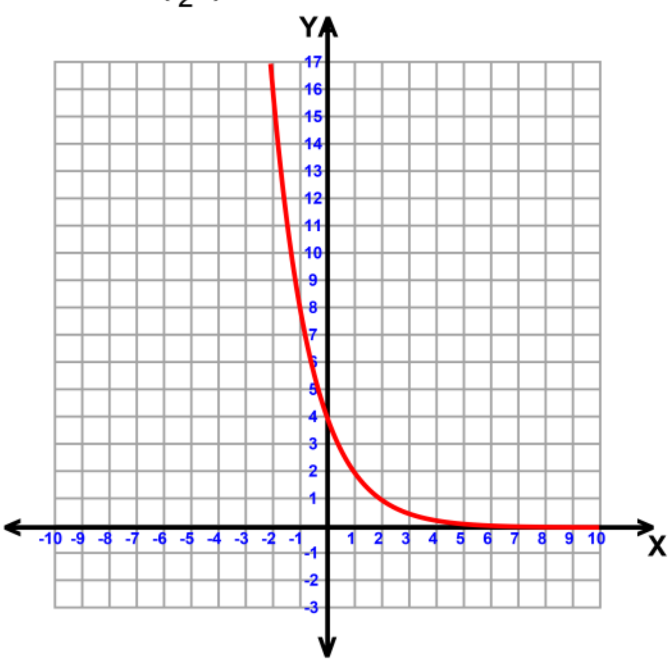
5) $y = 2 \cdot 4^x$



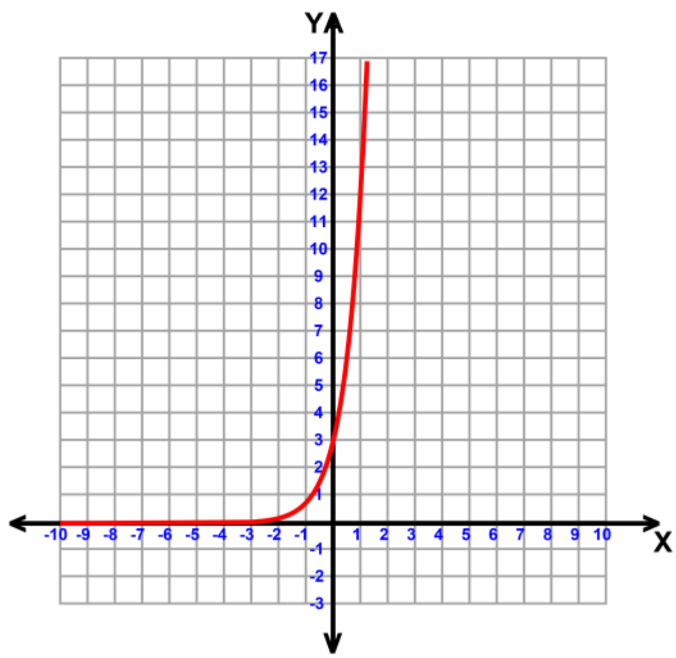
6) $y = 4 \cdot \left(\frac{1}{2}\right)^x$



7) $y = 4 \cdot \left(\frac{1}{2}\right)^x$



8) $y = 3 \cdot 4^x$



7.3 Solving Exponents

RECALL:

Rules of Exponents or Laws of Exponents	
Multiplication Rule	$a^x \times a^y = a^{x+y}$
Division Rule	$a^x \div a^y = a^{x-y}$
Power of a Power Rule	$\left(a^x\right)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Solving when the base is a variable x:

Example 1 Solve the following equation for x.

$$x^{\frac{3}{2}} = 125$$

Example 2 Solve the following equation for x.

$$2x^{\frac{3}{4}} = 54$$

Example 3 Solve the following equation for x.

$$(2x+1)^{\frac{2}{3}} = 4$$

When the variable is in the exponent:

- **convert each base to the same base**

Exponential Equations with Same Base

$a^x = a^y$ If and only if $x = y$

Examples:

$2^x = 64$
$\Rightarrow 2^x = 2^6$
$\Rightarrow x = 6$

$3^{2x} = 27^{x-4}$
$\Rightarrow 3^{2x} = \left(3^3\right)^{x-4}$
$\Rightarrow 2x = 3(x-4)$
$\Rightarrow 2x = 3x - 12$
$\Rightarrow -x = -12$
$\Rightarrow x = 12$

https://slideplayer.com/slide/6897597/

Solve by Equating Exponents

Solve $4^{3x} = 8^{x+1}$	Original Problem
$(2^2)^{3x} = (2^3)^{x+1}$	Rewrite with base 2
$2^{6x} = 2^{3x+3}$	Simplify exponents
$6x = 3x + 3$	Equate exponents
$x = 1$	Simplify

Goal 1: Solving Exponential Functions

Solving Exponential Equations:

- If possible, express both sides as powers of the same base

$$27(3^{x+1}) = 9^{2x-7}$$
$$3^3(3^{x+1}) = (3^2)^{2x-7}$$
$$3^{3+x+1} = 3^{2(2x-7)}$$
$$3^{x+4} = 3^{4x-14}$$

- Equate the exponents

$$x + 4 = 4x - 14$$

- Solve for variable

$$4 = 3x - 14$$
$$18 = 3x$$
$$\boxed{x = 6}$$

Solve: $\left(\frac{2}{3}\right)^{x+6} = \left(\frac{8}{27}\right)^{3x}$

1. Find a common base

$$\frac{8}{27} = \left(\frac{2}{3}\right)^3$$

$$\left(\frac{2}{3}\right)^{x+6} = \left[\left(\frac{2}{3}\right)^3\right]^{3x}$$

$$\left(\frac{2}{3}\right)^{x+6} = \left(\frac{2}{3}\right)^{9x}$$

2. Equate the exponents

$$x + 6 = 9x$$

$$x = \frac{3}{4}$$

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Example 1 Solve the following equation for x.

$$2^{3x-1} = 16$$

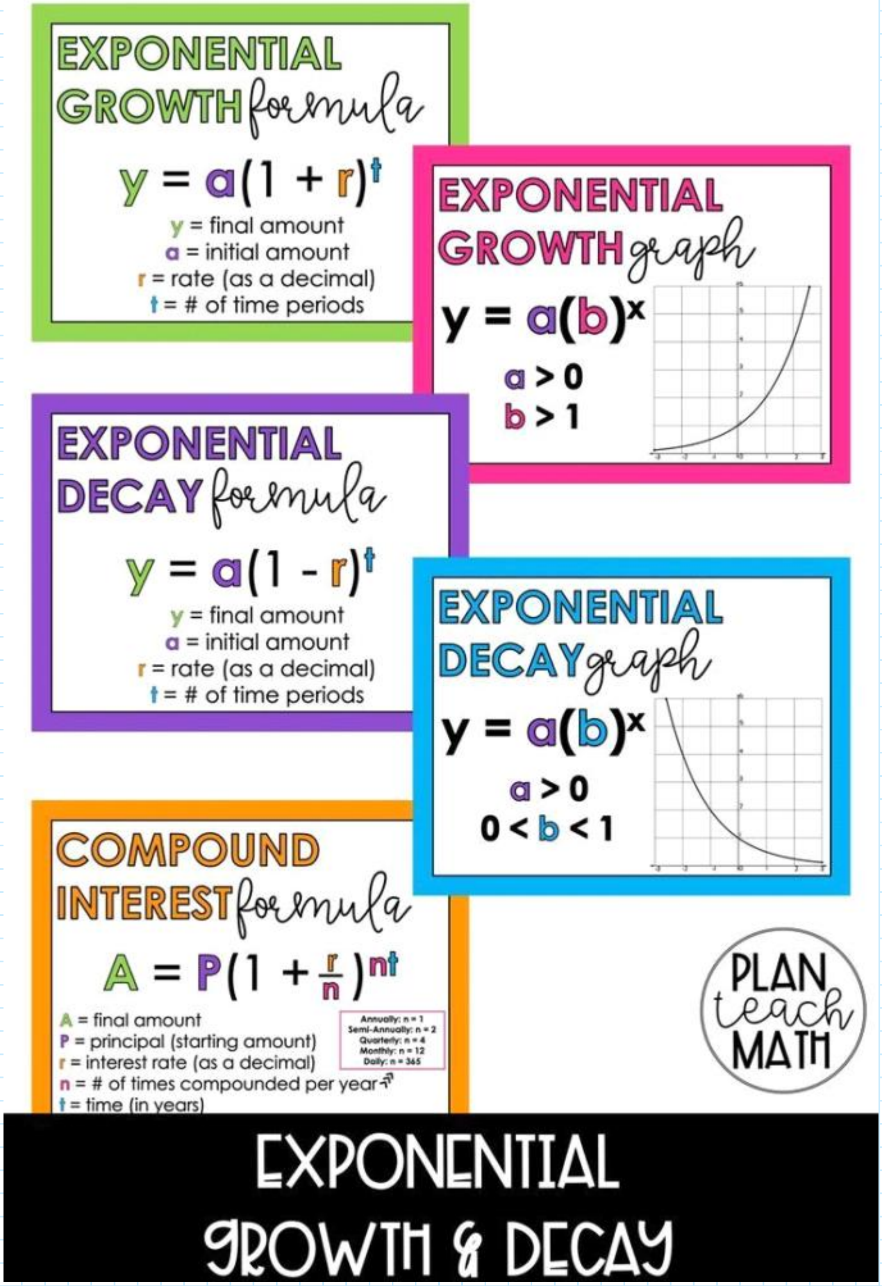
Example 2 Solve the following equation for x.

$$27^{2x-1} = 9^{x+2}$$

Example 3 Solve the following equation for x.

$$\frac{1}{343^{x-1}} = 49^{2x-1}$$

APPLICATIONS


EXPONENTIAL GROWTH formula
y = a(1 + r)^t
y = final amount
a = initial amount
r = rate (as a decimal)
t = # of time periods
EXPONENTIAL GROWTH graph
y = a(b)^x
a > 0
b > 1
EXPONENTIAL DECAY formula
y = a(1 - r)^t
y = final amount
a = initial amount
r = rate (as a decimal)
t = # of time periods
EXPONENTIAL DECAY graph
y = a(b)^x
a > 0
0 < b < 1
COMPOUND INTEREST formula
A = P(1 + r/n)^nt
A = final amount
P = principal (starting amount)
r = interest rate (as a decimal)
n = # of times compounded per year
t = time (in years)
Annually: n = 1
Semi-Annually: n = 2
Quarterly: n = 4
Monthly: n = 12
Daily: n = 365
PLAN teach MATH
EXPONENTIAL GROWTH & DECAY

Exponential growth and decay can be modeled using the formula: $A = A_o(b)^{\frac{t}{n}}$

A = final amount
A_o = initial amount
b = base which is the factor of change (growth or decay factor)
t = time elapsed
n = interval of time for growth or decay

Compound Interest: $A = P(1+i)^{tn}$ OR $A = P\left(1+\frac{r}{n}\right)^{tn}$

P = Principal (initial amount)
i = Interest rate divided by the number of times compounded per year
n = number of compounding periods per year

Richter Scales for Earthquakes:

$$I = I_{\circ}(10)^{R_{high} - R_{low}}$$

I = the intensity between the two Richter scale magnitudes
R = difference in Richter scale magnitudes where 1 unit represents a 10-fold increase or decrease
in magnitude.

Decibel Scale:

$$I = I_{\circ}(10)^{\frac{Db_{high} - Db_{low}}{10}}$$

I = the intensity of sound between the two decibel levels
Db = difference in decibel scale levels where 10 units represent a 10-fold increase or decrease in
decibel level.

pH Scale:

$$I = I_{\circ}(10)^{pH_{high} - pH_{low}}$$

I = the level of acidity or basicity between the two pH values
pH = difference in values on pH scale where 1 unit represents a 10-fold increase or decrease in pH
level (translates to change in acidity of a solution; either more or less acidic or basic).

7.3 Solving Exponential Equations

Remember the rules for working with exponents:

product rule: $a^m a^n = a^{m+n}$ add exponents (bases are the same)
ex: $2^6 \cdot 2^{10} = 2^{16}$ $2^6 \cdot 3^{10} \neq \ldots$ answer answer + multiply

power of a power: $(a^m)^n = a^{mn}$ multiply exponents
ex: $(3^2)^5 = 3^{10}$ $(4^{2x})^{x+1} = 4^{2x^2+2x}$

distributive: $(ab)^m = a^m b^m$ distribute exponent to inside terms ✱ only when terms multiply
ex $(x^2y)^3 = x^6y^3$ NOT $(x+y)^2 \neq x^2+y^2$ $= (x+y)(x+y)$ $x^2+2xy+y^2$

": $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
ex: $\left(\frac{3x}{2y}\right)^2 = \frac{9x^2}{4y^2}$

quotient law: $\frac{a^m}{a^n} = a^{m-n}$ subtract exponent (same base)
ex: $\frac{x^4}{x} = x^{4-1} = x^3$

rational exponent law: $a^{\frac{m}{n}} = \sqrt[n]{a}^{\,m}$ OR $\sqrt[n]{a^m}$ (exponent, index)
fraction exponent changes into radical where denominator is the root number (index)
ex: $16^{\frac{3}{4}} = \sqrt[4]{16}^{\,3} = 2^3 \Rightarrow \boxed{8}$

Exponential equations are ones where the ***variable is in the exponent***. We can solve these equations by
- Writing the left side of the equation and the right side of the equation so they each use the same base.
- Then, we use the fact that if $a^x = a^y$, it forces $x = y$, to finish solving the equation.

when the bases are the same — the exponents are equal

ex: $2^x = 2^5$ $\boxed{x=5}$

$2^x \cdot 2^3 = 2^4$ can't cancel

7.3 Ex 1 changing the base p.360

Your Turn

Write each expression as a power with base 2.

a) 4^3 **b)** $\frac{1}{8}$ **c)** $8^{\frac{2}{3}}(\sqrt{16})^3$

$(2^2)^3$ power of power $= 8^{-1}$ negative $(2^3)^{2/3}$ $\frac{3}{2}$ $3 \times \frac{2}{3} = 2$

$(2^2)^3$ power of a power (multiply exponents)

$= 2^6$

$= 8^{-1}$ negative exponent to reciprocal the base

$= (2^3)^{-1}$

$= 2^{-3}$

$= (2^3)^{2/3} \; 16^{\frac{3}{2}}$ $\quad 3 \times \frac{2}{3} = 2$

$= 2^2 \, (2^4)^{3/2}$ $\quad 4 \times \frac{3}{2} = 6$ power law = multiply exponents.

$= 2^2 \, 2^6$ product law = add exponents

$= 2^{2+6}$

$= 2^8$

Example

$$8^{4x-1} = \left(\frac{1}{2}\right)^{x+5}$$

common base = 2

$(2^3)^{4x-1} = (2^{-1})^{x+5}$

$2^{12x-3} = 2^{-x-5}$

$12x - 3 = -x - 5$

$13x = -2$

$x = -\frac{2}{13}$

To Try

1. Rewrite the expressions so they have the same base, then solve the equation.

a) $\left(\frac{1}{25}\right)^{4x} = (125)^{3x+2}$

$(5^{-2})^{4x} = (5^3)^{3x+2}$

$5^{-8x} = 5^{9x+6}$

$-8x = 9x + 6$

$-17x = 6$

$x = -\frac{6}{17}$

* DO #7 ch7 assign

b) $(8)^{2x+7} = 16^{4x+2}$

$(2^3)^{2x+7} = (2^4)^{4x+2}$

$2^{6x+21} = 2^{16x+8}$

$6x + 21 = 16x + 8$

$-10x = -13$

$x = \frac{13}{10}$

c) $16^{3x} = 8^{3x-1}64^x$

$(2^4)^{3x} = (2^3)^{3x-1}(2^6)^x$

$2^{12x} = 2^{(9x-3} \cdot 2^{+6x)}$ ← add exponents

$2^{12x} = 2^{15x-3}$

$12x = 15x - 3$

$-3x = -3$

$x = 1$

t = unknown   principle = P   rate = r

2.For how long does one need to invest $2000 in an account that earns 6.1% compounded quarterly, before it increases in value to $2500?

n = 4   Final Amount = A

$A = P(1+i)^n \rightarrow A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow 2500 = 2000\left(1+\frac{0.061}{4}\right)^{4t}$

P = principal amount deposited

i = interest rate per compounding period, in decimal form $= \frac{r}{n}$

n = number of compounding periods → nt

\# of times compounded per year:

- annually → n = 1
- semi-annually → n = 2
- quarterly → n = 4
- monthly → n = 12
- bi-weekly → n = 26
- semi-monthly → n = 24
- weekly → n = 52
- daily → n = 365

$\frac{5}{4} = (1.01525)^{4t}$

$1.25 = 1.01525^{4t}$

common base? NO

∴ need logs to solve → ch8

3. The population of a town triples every 6 years. If 4000 people lived there in 2009, how many will be in the town 2030? (Round down to the nearest whole person.)

b = 3, p = 6, $A_0 = 4000$

t = 2030 − 2009 = 21 yr.

$A = 4000(3)^{\frac{21}{6}}$

A = 187061 people.

7.3 Practice

Exponential
Equations...

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Exponential Equations Not Requiring Logarithms

Date________________ Period____

Solve each equation.

1) $4^{2x+3} = 1$

2) $5^{3-2x} = 5^{-x}$

3) $3^{1-2x} = 243$

4) $3^{2a} = 3^{-a}$

5) $4^{3x-2} = 1$

6) $4^{2p} = 4^{-2p-1}$

7) $6^{-2a} = 6^{2-3a}$

8) $2^{2x+2} = 2^{3x}$

9) $6^{3m} \cdot 6^{-m} = 6^{-2m}$

10) $\frac{2^x}{2^x} = 2^{-2x}$

11) $10^{-3x} \cdot 10^{x} = \frac{1}{10}$

12) $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

13) $4^{-2x} \cdot 4^{x} = 64$

14) $6^{-2x} \cdot 6^{-x} = \frac{1}{216}$

15) $2^{x} \cdot \frac{1}{32} = 32$

16) $2^{-3p} \cdot 2^{2p} = 2^{2p}$

17) $64 \cdot 16^{-3x} = 16^{3x-2}$

18) $\frac{81^{3n+2}}{243^{-n}} = 3^{4}$

19) $81 \cdot 9^{-2b-2} = 27$

20) $9^{-3x} \cdot 9^{x} = 27$

21) $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

22) $243^{k+2} \cdot 9^{2k-1} = 9$

23) $16^{r} \cdot 64^{3-3r} = 64$

24) $16^{2p-3} \cdot 4^{-2p} = 2^{4}$

Kuta Software - Infinite Algebra 2 Name____________________

Exponential Equations Not Requiring Logarithms

Date________________ Period____

Solve each equation.

1) $4^{2x+3} = 1$

$\left\{-\frac{3}{2}\right\}$

2) $5^{3-2x} = 5^{-x}$

$\{3\}$

3) $3^{1-2x} = 243$

$\{-2\}$

4) $3^{2a} = 3^{-a}$

$\{0\}$

5) $4^{3x-2} = 1$

$\left\{\frac{2}{3}\right\}$

6) $4^{2p} = 4^{-2p-1}$

$\left\{-\frac{1}{4}\right\}$

7) $6^{-2a} = 6^{2-3a}$

$\{2\}$

8) $2^{2x+2} = 2^{3x}$

$\{2\}$

9) $6^{3m} \cdot 6^{-m} = 6^{-2m}$

$\{0\}$

10) $\frac{2^x}{2^x} = 2^{-2x}$

$\{0\}$

11) $10^{-3x} \cdot 10^x = \frac{1}{10}$

$\left\{\frac{1}{2}\right\}$

12) $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

$\left\{-\frac{2}{3}\right\}$

13) $4^{-2x} \cdot 4^{x} = 64$

$\{-3\}$

14) $6^{-2x} \cdot 6^{-x} = \frac{1}{216}$

$\{1\}$

15) $2^{x} \cdot \frac{1}{32} = 32$

$\{10\}$

16) $2^{-3p} \cdot 2^{2p} = 2^{2p}$

$\{0\}$

17) $64 \cdot 16^{-3x} = 16^{3x-2}$

$\left\{\frac{7}{12}\right\}$

18) $\frac{81^{3n+2}}{243^{-n}} = 3^{4}$

$\left\{-\frac{4}{17}\right\}$

19) $81 \cdot 9^{-2b-2} = 27$

$\left\{-\frac{3}{4}\right\}$

20) $9^{-3x} \cdot 9^{x} = 27$

$\left\{-\frac{3}{4}\right\}$

21) $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

$\left\{-\frac{1}{6}\right\}$

22) $243^{k+2} \cdot 9^{2k-1} = 9$

$\left\{-\frac{2}{3}\right\}$

23) $16^{r} \cdot 64^{3-3r} = 64$

$\left\{\frac{6}{7}\right\}$

24) $16^{2p-3} \cdot 4^{-2p} = 2^{4}$

$\{4\}$

8.1: Understanding Logarithms

What is a log?
A logarithm is an exponent.

$$\log_b(a) = c \iff b^c = a$$

$\log_b a = c$

$a = b^c$

$b^c = a$

log

$\log_b a = c$

Logarithmic Form		Exponential Form
$\log_3 x = 5$	➡	$3^5 = x$

Both forms use the same base.

The logarithm is equal to the exponent.

Chapter 8: Logarithmic Functions

8.1 Understanding Logarithms

A logarithm tells how many copies of one number we need to multiply together, to create a different number.

For example:
How many 4's do we have to multiply together to get 64?

$4\times4\times4=64$, which shows we have to multiply 3 of the "4's" to produce 64

This tells us the ***logarithm*** is 3.

Because $4\times4\times4=64$, we can say: $\log_4(64)=3$

- we can read this as "The logarithm base 4 of 64 is equal to 3"
- we can shorten it a bit, and say "log base 4 of 64 equals 3"

$$\log_4(64)$$

arg

A logarithm tells us how many copies of the BASE we need to multiply together, to create the ARGUMENT – in other words, the logarithm is the ***exponent*** we raise the base to, in order to produce the argument.

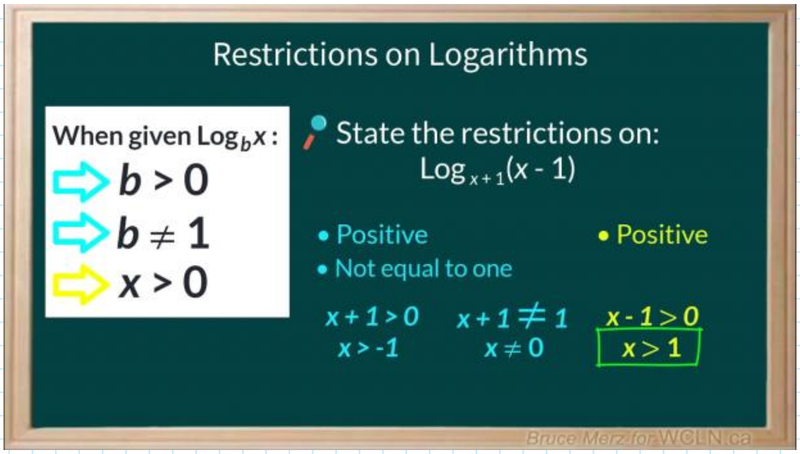
Try These

1. $\log_4(16)=$
2. $\log_3(27)=$
3. $\log_{0.5}(0.5)=$
4. $\log_a(a^7)=$
5. $\log_2(2^{-3})=$
6. $\log_1 7=$
7. $\log_5\left(\frac{1}{25}\right)=$
8. $\log_3 0=$
9. $\log_2(-4)=$
10. $\log_6(\sqrt{6})=$
11. $\log_2\left(\sqrt[3]{2^4}\right)=$
12. $\log_8(1)=$

For a logarithm to make sense, we need the argument and the base to obey these restrictions:

argument **base**

Logarithms have restrictions (non-permissible values = NPVs)


Restrictions on Logarithms
When given $Log_b x$:
$b > 0$
$b \neq 1$
$x > 0$
State the restrictions on:
$Log_{x+1}(x - 1)$
• Positive
• Not equal to one
• Positive
$x + 1 > 0$
$x > -1$
$x + 1 \neq 1$
$x \neq 0$
$x - 1 > 0$
$x > 1$
Bruce Merz for WCLN.ca

Notation

log base 10 is called **COMMON log** — $\log_{10} x$ *is written* $\log x$

log base *e* is called **NATURAL log** — $\log_e x$ *is written* $\ln x$

The number *e* is a very important irrational number. Its decimal expansion starts out:

$$e \approx \mathbf{2.71828182845904523536028747} \mathbf{13\ldots}$$

https://www.popularmechanics.com/science/math/a24383/mathematical-constant-e/

Changing Form

Exponents and logarithms are closely connected. Look at these two equations:

$\log_4 64 = 3$ (logarithmic form) and $4^3 = 64$ (exponential form)

Boot base.

Both equations show the relationship between the numbers 4, 3 and 64. We need to know how to change equations from one form to the other, as in some questions one form is better than the other.

To Try

1. Change form.

a) $\log_6 216 = 3$

$6^3 = 216$

b) $\log_p q = r$

$q = p^r$ or $p^r = q$

c) $\log 1000 = 3$

d) $7^2 = 49$

$\log_7 49 = 2$

e) $5^{x+y} = a$

$\log_5 a = x + y$

f) $49^{1/2} = 7$

2. Solve for *x*.

a) $\log_2(x-1) = 3$

Change to exp. form

$x - 1 = 2^3$

$x - 1 = 8$

+1

$x = 9$

Restriction: $\log x$, $x > 0$

$x - 1 > 0$

$x > 1$

b) $\log_6 x = -2$

c) $\log_x 8 = 3$

d) $\ln x = 2$

e) $\log_2(\log_9 x) = -1$

f) $4^{\log_4 7} = x$

But there is a short-cut in some cases!

Try solving these:

a) $\log_x 27 = 3$

b) $\log_2 (2x-5) = 4$

c) $\log_{(x+1)} (2x-1) = 2$

Try to evaluate these logs:

a) $\log_2 64$

b) $\log_4 0.0625$

c) $\log_{\frac{1}{2}} 32$

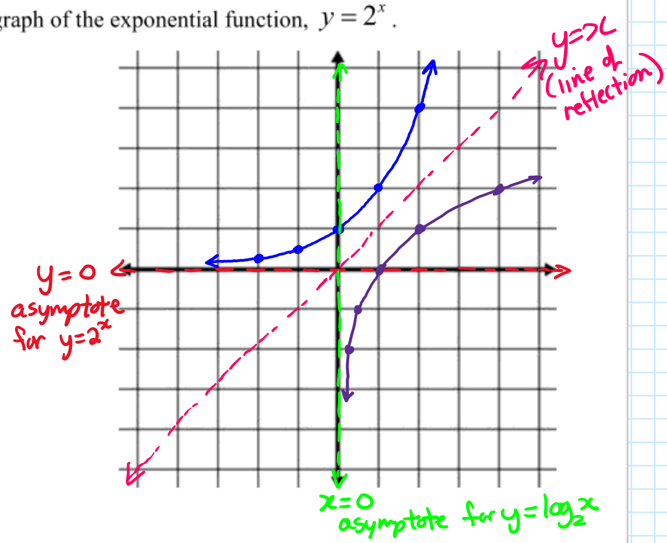
d) $\log_{1.2} 223.87$

Graphing an Exponential Function and its Inverse

a) Fill in the table below, and sketch the graph of the exponential function, $y = 2^x$.

x	y
−2	1/4
−1	1/2
0	1
1	2
2	4

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
$2^0 = 1$
$2^1 = 2$
$2^2 = 4$



b) Identify the following:

domain : $\{x \mid x \in \mathbb{R}\}$

range : $\{y \mid y > 0, y \in \mathbb{R}\}$

asymptote equation $y = 0$

x-intercept, if it exists none

y-intercept, if it exists $(0,1)$

c) Give the equation of the ***inverse*** of: $y = 2^x$. Inverse's equation is: $y = \log_2 x$

☆ same base

d) For the equation of the inverse that you found in part c), complete the table at right and sketch the graph on the grid above.

switch x + y from $y = 2^x$ graph (table)

x	y
1/4	−2
1/2	−1
1	0
2	1
4	2

e) For the inverse graph, what are its:

domain $\{x \mid x > 0, x \in \mathbb{R}\}$

range $\{y \mid y \in \mathbb{R}\}$

asymptote equation $x = 0$

x-intercept, if it exists $(1,0)$

y-intercept, if it exists none

f) Rewrite the inverse equation from part c) in logarithmic form: $y = \log_2 x$

Conclusions:

$\log_a\left(a^x\right) =$ $\qquad\qquad$ $a^{\log_a x} =$

Graphing a Logarithmic Function

Graphing a logarithmic function is the same as graphing the inverse of the exponential function with the same base.

Recall: to graph the inverse, switch the x and y for each coordinate and plot (the graph reflects over the line y=x and the domain and range also inverse)

Ex. Graph the function $y = \log_2 x$

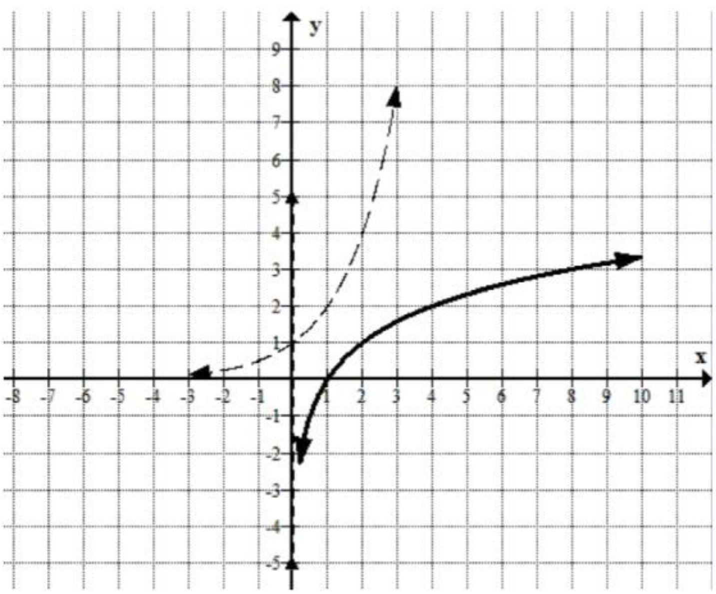
1. First determine the points on the function $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

2. Inverse each coordinate and the asymptote to become $x = 0$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

3. Graph the points.



Try graphing the following in the same steps as above:

a) $y = \log_3 x$ (base 3)

① use coordinates from $y=3^x$ & inverse them for $y=\log_3 x$

inverse

$y=3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

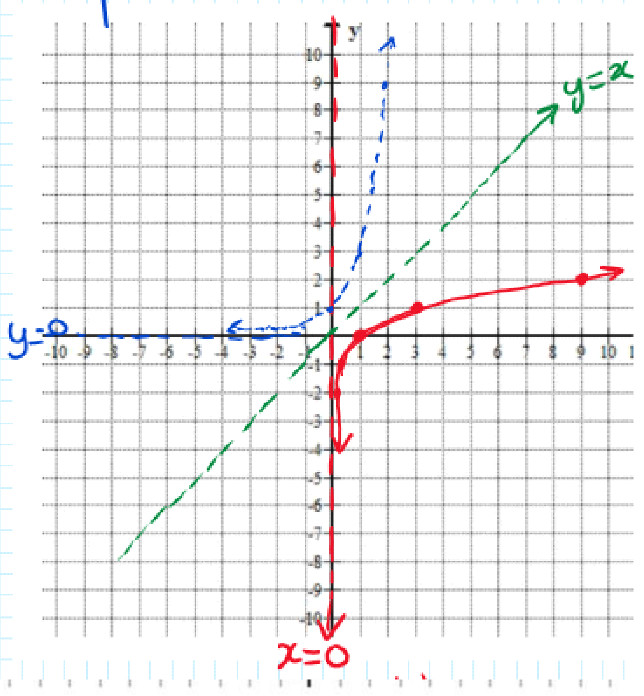
$3^{-1} = \frac{1}{3}$

$3^0 = 1$

$3^1 = 3$ $3^2 = 9$

$y=\log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2



b) $y = \log_{\left(\frac{1}{2}\right)} x$ (base $\frac{1}{2}$)

exponential function has the same base ∴ $y=\left(\frac{1}{2}\right)^x$

① table for $y=\left(\frac{1}{2}\right)^x$

x	y
-2	4
-1	2
0	1
1	1/2
2	1/4

$\left(\frac{1}{2}\right)^{-2} = 2^2$

$\left(\frac{1}{2}\right)^{-1}$

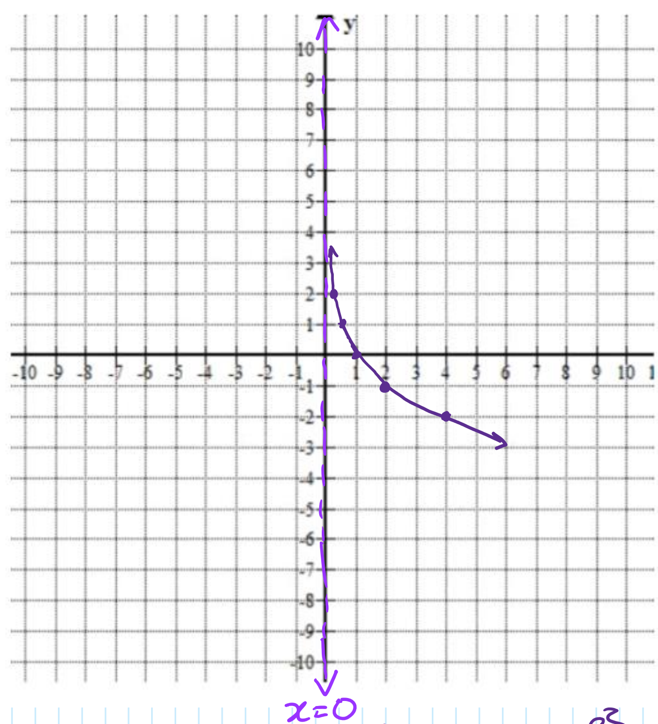
$\left(\frac{1}{2}\right)^0$

$\left(\frac{1}{2}\right)^1$

$\left(\frac{1}{2}\right)^2$

② inverse for log graph $y=\log_{\frac{1}{2}} x$

x	y
4	-2
2	-1
1	0
1/2	1
1/4	2



$\{x \mid x>0, x\in\mathbb{R}\}$

$\{y \mid y\in\mathbb{R}\}$.

x-int $(1,0)$