Class_17 June 1 - Logarithmic Graphs

Thursday, June 1, 2023 8:26 AM

Tonight's Class:

- **Warm-up at whiteboards - Chapter 7 Review. Skip #3b**
- **8.1 Understanding Logarithms (continued)**
- **8.2 Transforming Log Graphs**

Notation

log base 10 is called **COMMON log** — $\log_{10} x$ *is written* $\log x$

log base e is called **NATURAL log** — $\log_e x$ *is written* $\ln x$

The number e is a very important irrational number. Its decimal expansion starts out:

$e \approx 2.71828182845904523536028747l3\ldots$

https://www.popularmechanics.com/science/math/a24383/mathematical-constant-e/

Changing Form

Exponents and logarithms are closely connected. Look at these two equations:

$\log_4 64 = 3$ (**logarithmic form**) and $4^3 = 64$ (**exponential form**)

Boot base.

Both equations show the relationship between the numbers 4, 3 and 64. We need to know how to change equations from one form to the other, as in some questions one form is better than the other.

$\log_2 8 = $ ~~a~~ number

base — argument? — exponent need, to make base become the argument

To Try

1. Change form.

a) $\log_6 216 = 3$ — $6^3 = 216$

b) $\log_p q = r$ — $q = p^r$ or $p^r = q$

c) $\log 1000 = 3$ — $10^3 = 1000$

d) $7^2 = 49$ — $\log_7 49 = 2$

e) $5^{x+y} = a$ — $\log_5 a = x+y$

f) $49^{1/2} = 7$ — $\log_{49} 7 = \frac{1}{2}$

$\log_6 36 = 2$

$\log_2 \frac{1}{4} = -2$

$\log_7 7 = 1$

$2^{\square} = \frac{1}{4}$

$\frac{1}{2^2} = 2^{-2} = \frac{1}{4}$

2. Solve for x.

a) $\log_2(x-1) = 3$

Change to exp. form

$x - 1 = 2^3$

$x - 1 = 8$ (+1)

$\boxed{x = 9}$

Restriction: $\log x$, $x > 0$; $x - 1 > 0$; $x > 1$

b) $\log_6 x = -2$

Change to exponential form

$6^{-2} = x$

$x = \frac{1}{6^2} = \boxed{\frac{1}{36}}$

c) $\log_x 8 = 3$

$(x^3)^{1/3} = (8)^{1/3}$

$x = 2$

d) $\ln x = 2$

means $\log_e$ "natural log"

$e^2 = x$

$x \doteq 7.39$ (using calculator)

e) $\log_2(\log_9 x) = -1$

$2^{-1} = \log_9 x$

$\frac{1}{2} = \log_9 x$

$9^{1/2} = x$, $x = \sqrt{9}$, $\boxed{x = 3}$

f) $4^{\log_4 7} = x$

$\log_4 x = \log_4 7$

$\Rightarrow \boxed{x = 7}$

Try solving these:

a) $\log_x 27 = 3$

$(x^3)^{1/3} = (27)^{1/3}$

$x = (27)^{1/3}$

$\boxed{x = 3}$

b) $\log_2(2x-5) = 4$

$2^4 = 2x - 5$

$16 = 2x - 5$ (+5, +5)

$\frac{21}{2} = \frac{2x}{2}$

$\boxed{x = \frac{21}{2}}$

check that it's okay: $2\left(\frac{21}{2}\right) - 5 = 21 - 5 = 16$ — argument. This is > 0, so it's fine.

c) $\log_{(x+1)}(2x-1) = 2$

$(x+1)^2 = 2x - 1$

$(x+1)(x+1) = 2x - 1$

$x^2 + 1x + 1x + 1 - 2x + 1 = 0$

Try to evaluate these logs:

a) $\log_2 64 = x$

$2^x = 64$

$\boxed{x = 6}$

b) $\log_4 0.0625 = x$

$4^x = 0.0625$

$4^x = 4^{-2}$

$\Rightarrow \boxed{x = -2}$

c) $\log_{\frac{1}{2}} 32 = x$

$\left(\frac{1}{2}\right)^x = 32$

$2^{-x} = 32$

$2^{-x} = 2^5$

$\frac{-x}{-1} = \frac{5}{-1}$

$\boxed{x = -5}$

d) $\log_{1.2} 223.87$

$\log_x a = c$

$x > 0$

$x \neq 1$

argument > 0

Sneak Preview

$\log_{1.2} 223.87$

$$(x+1)(x+1) = 2x-1$$
$$x^2 + 1x + 1x + 1 - 2x + 1 = 0$$
$$x^2 + \cancel{2x} + 1 - \cancel{2x} + 1 = 0$$
$$x^2 + 2 = 0$$
$$x^2 = -2 \Rightarrow \cancel{x = \text{nothing}}\,!$$

$\boxed{\text{no solution}}$

$$\log_{1.2} 223.87$$
$$= \frac{\log 223.87}{\log 1.2} \doteq 29.67$$

Using Change of Base Formula

When we solve a logarithmic equation, we must make sure that the answer is okay.

- **We substitute the x-value into the argument.**
- **If it makes the argument 0 or negative, we must REJECT that answer.**

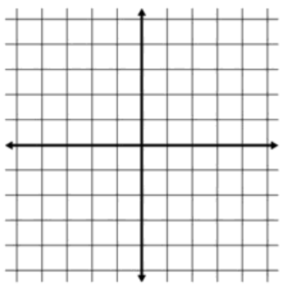
Answers that don't obey restrictions are called "extraneous roots"

DONE in yesterday's class

Graphing an Exponential Function and its Inverse

a) Fill in the table below, and sketch the graph of the exponential function, $y = 2^x$.

x	y



b) Identify the following:
domain
range
asymptote equation
x-intercept, if it exists
y-intercept, if it exists

c) Give the equation of the ***inverse*** of: $y = 2^x$. Inverse's equation is: ____________

d) For the equation of the inverse that you found in part c), complete the table at right and sketch the graph on the grid above.

x	y

e) For the inverse graph, what are its:
domain
range
asymptote equation
x-intercept, if it exists
y-intercept, if it exists

f) Rewrite the inverse equation from part c) in logarithmic form:

Conclusions:

$\log_a\left(a^x\right) =$ $\quad\quad$ $a^{\log_a x} =$

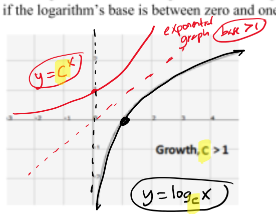
Log Practice worksheet, #1-5 only

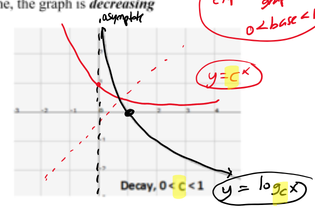
8.2 Transformations of Logarithmic Functions

The graphs of logarithmic functions can be grouped into two categories:

- if the logarithm's base is larger than one, the graph is ***increasing***
- if the logarithm's base is between zero and one, the graph is ***decreasing***

exponential graph 0 < base < 1





What characteristics are the *same* for all untransformed logarithmic graphs?

domain $\{x \mid x>0, x\in\mathbb{R}\}$ x-intercept $(1,0)$

range $\{y \mid y\in\mathbb{R}\}$ asymptote (and its equation) $\boxed{x=0}$ on the y-axis

Predict what will happen to the graph of $y=\log_3 x$ when each of the following changes is made to the equation:

$y=\log_3(x)-5$ down 5

$y=-4\log_3 x$ ~~VE by −4~~ VE by 4, and reflected across the x-axis

$y=\log_3\left(-\frac{2}{5}(x+3)\right)$ $\underbrace{\text{reflect across y-axis, HE by } \frac{5}{2}}_{\text{HE by } -\frac{5}{2}}$, 3 left

Horizontal stretch by a factor of $\frac{1}{b}$

Vertical stretch by a factor of a

Horizontal translation

Vertical translation

$$y=a\log_c b(x-h)+k$$

If b < 0 then there is a reflection over the y-axis (horizontal reflection)
If a < 0 then there is a reflection over the x-axis (vertical reflection)

When a log equation is transformed, here's the an easy way to find its domain and vertical asymptote equation:

To find the domain of a logarithmic graph

- **Set the ARGUMENT > 0, and isolate x**

The vertical asymptote equation is in the form x = c, where "c" is the number that appears in the domain.

Example
Without graphing, find the following characteristics for this equation's graph.

$y=\log_4(-2x+16)-3$

- Domain
- Range → $\{y \mid y\in\mathbb{R}\}$
- Asymptote equation $\boxed{x=8}$
- *x*-intercept
- *y*-intercept

Set argument > 0

$-2x+16>0$

$\frac{-2x}{-2} > \frac{-16}{-2}$ (reverse inequality when dividing by a negative)

$x<8$

$-2x+16>0$
$+2x \qquad +2x$
$\frac{16}{2} > \frac{2x}{2}$
$8>x$ or $x<8$

x-intercept, let $y=0$ and solve.

$0=\log_4(-2x+16)-3$
$+3 \qquad +3$

$3=\log_4(-2x+16)$

$(-24,0)$

$4^3=-2x+16$

$(-24, 0)$

$$3 = \log_4(-2x+16)$$
$$4^3 = -2x+16$$
$$64 = -2x+16$$
$$\frac{48}{-2} = \frac{-2x}{-2} \qquad x = -24$$

$\underline{y\text{-int}}$, let $x = 0$

$$y = \log_4(-2x+16) - 3$$
$$y = \log_4(-2(0)+16) - 3$$
$$y = \log_4(16) - 3$$
$$y = 2 - 3$$
$$y = -1$$

$(0, -1)$

Graph:

$$y = \log_4(-2x+16) - 3$$

$\boxed{y = 4^x}$

x	y
-1	$4^{-1} = \frac{1}{4}$
0	$4^0 = 1$
1	$4^1 = 4$
2	$4^2 = 16$

$\boxed{y = \log_4 x}$

x	y
$\frac{1}{4}$	-1
1	0
4	1
16	2

1) similar to $y = \log_4 x$

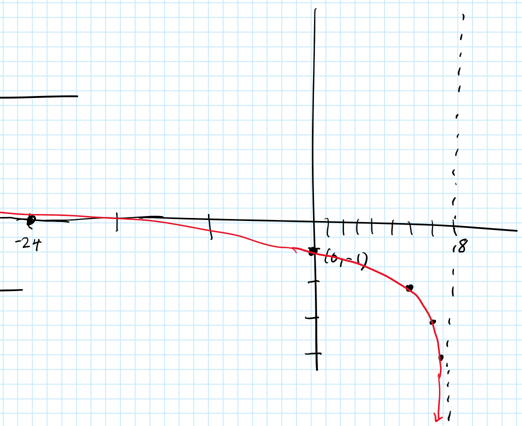
2) make table for its inverse, $y = 4^x$

3) figure out the mapping
factor first!!
$$y = \log_4(-2(x-8)) - 3$$
$$(x, y) \rightarrow \left(-\tfrac{1}{2}x + 8,\ y - 3\right)$$

4) Perform the mapping on the BASE table points.

$\boxed{y = \log_4(-2x+16) - 3}$

$-\frac{1}{2}x + 8$	$y - 3$
$(-\frac{1}{2})(\frac{1}{4}) + 8 = -\frac{1}{8} + 8 = \frac{63}{8} = 7\frac{7}{8}$	$-1 - 3 = -4$
$(-\frac{1}{2})(1) + 8$; $-\frac{1}{2} + 8$; $7\frac{1}{2}$	-3
$(-\frac{1}{2})(4) + 8$; 6	$1 - 3 = -2$
$(-\frac{1}{2})(16) + 8 = 0$	$2 - 3 = -1$



WB Log Transformations

Your Turn

a) Use transformations to sketch the graph of the function $y = 2\log_3(-x + 1)$. FACTOR $y = 2\log_3(-1(x-1))$

b) Identify the following characteristics.

i) the equation of the asymptote **ii)** the domain and range

iii) the y-intercept, if it exists **iv)** the x-intercept, if it exists

Exponential function it's related to: $y = 3^x$

x	y
-1	$\frac{1}{3}$
0	1
1	3
2	9

Base function: $y = \log_3 x$

x	y
$\frac{1}{3}$	-1
1	0
3	1
9	2

$$y = 2\log_3(-x+1)$$

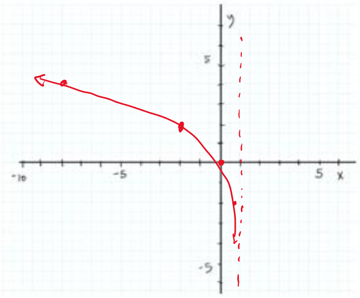
Base function

$$y = \log_3 x$$

$y = 2\log_3(-x+1)$

Base function

$y = \log_3 x$



Transformations and mapping notation

$(x,y) \rightarrow (-x+1, 2y)$

Domain $-x+1>0$

$\frac{-x}{-1} > \frac{-1}{-1}$

$\{x \mid x<1, x \in \mathbb{R}\}$

Range $\{y \mid y \in \mathbb{R}\}$

Vertical asymptote equation

$x = 1$

1	3
2	9

3	1
9	2

mapping:

$(x,y) \rightarrow (-x+1, 2y)$

$-x+1$	$2y$
$-\frac{1}{3}+1 \cdot \frac{3}{3}$ $= \frac{2}{3}$	$2(-1)$ $=-2$
$-1+1$ $=0$	$2(0)$ $=0$
$-3+1$ $=-2$	$2(1)$ $=2$
$-9+1$ $=-8$	$2(2)$ $=4$

y-int, let $x=0$

$y = 2\log_3(-0+1)$

$= 2\log_3(1)$

$= 2(0)$

$= 0$

$(0,0)$

x-int, let $y=0$

$\frac{0}{2} = \frac{2\log_3(x+1)}{2}$

$0 = \log_3(x+1)$

$3^0 = x+1$

$1 = x+1$

$x=0$

$(0,0)$

For each of the following find:

- Domain
- Range
- Asymptote equation
- *x*-intercept
- *y*-intercept

1. $y = \log_2(x-7)-5$

domain $x-7>0$

$x>7$ $\{x \mid x>7, x \in \mathbb{R}\}$

range: $\{y \mid y \in \mathbb{R}\}$

asymptote $x=7$

x-int $0 = \log_2(x-7)-5$

$5 = \log_2(x-7)$

$2^5 = x-7$

$32 = x-7$

$39 = x$

$(39,0)$

y-int

$y = \log_2(0-7)-5$

$y = \log_2(-7)-5$

impossible to do!

$\boxed{\text{no y-int}}$

2. $y = \log_3(5x+3)-4$

domain $5x+3>0$

$5x > -3$

$\{x \mid x > -\frac{3}{5}, x \in \mathbb{R}\}$

range $\{y \mid y \in \mathbb{R}\}$

asymptote $x = -\frac{3}{5}$

x-int, let $y=0$

$0 = \log_3(5x+3)-4$

$4 = \log_3(5x+3)$

$3^4 = 5x+3$

$81 = 5x+3$

$\frac{78}{5} = \frac{5x}{5}$

$\boxed{x = \frac{78}{5}}$

y-int, let $x=0$

$y = \log_3(5(0)+3)-4$

$y = \log_3(3)-4$

$y = 1-4$

$y=-3$

$(0,-3)$

For next class

Complete the Chapter 7 Hand-in
Prepare for Test 5 (on sections 6.4, 7.1-7.3, 8.1-8.2)
Start working on the Chapter 8 Hand-in. Should now be okay to do #1-7