Class_18 June 5 Log Laws

Plan For Today:

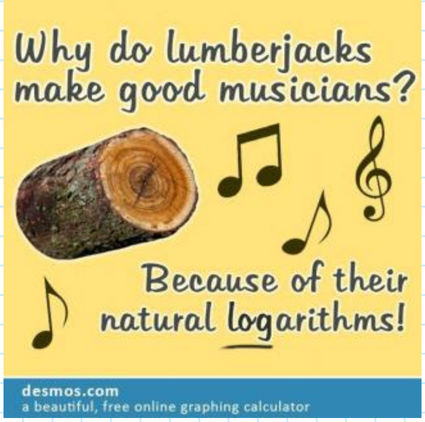
Hand-in Chapter 7 Assignment for Checking Completion

1. Question about anything from last week? 6.4, 7.1-7.3, 8.1-8.2
   - DO TEST 5

2. Continue Chapter 8: Logarithmic Functions
   - ✓ 8.1: Understanding Logarithms
   - ✓ 8.2: Transformations of Logarithmic Functions
   - ❖ **8.3: Laws of Logarithms**
   - ❖ 8.4: Logarithmic & Exponential Functions

5. Work on practice questions from Textbook

Page 400: #1-5, 8-10, 13bc, 16bc



Plan Going Forward:

1. Finish working through extra practice & textbook questions from 8.3 and continue working on the Ch. 8 Assignment.

2. You will go over 8.3 practice and start 8.4 solving log equations tomorrow.

- ❖ CHAPTER 8 ASSIGNMENT DUE THURSDAY, JUNE 8TH OR MONDAY, JUNE 12TH
- ❖ TEST 6 ON 8.2-9.2 ON MONDAY, JUNE 12TH

Please let us know if you have any questions or concerns about your progress in this course. The notes from today will be posted at egolfmath.weebly.com after class.
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Test 5 Review 6.4,Ch7,8.1,8.2

Ex: $\log_5(2x-3)=2$.

R

$2x-3>0$

$x>\frac{3}{2}$

$2x-3=5^2$

$2x-3=25$

$+3$

$\frac{2x}{2}=\frac{28}{2}$

$\boxed{x=14}$

$\log_x(x+6)=2$

$x^2=x+6$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$\boxed{x=3 \quad x=-2}$

R

$x+6>0$

$x>-6$

C_19 WB Log Patterns

Sunday, November 3, 2019 3:12 PM

Logarithms – Investigation

Part I:

Evaluate the expressions on the left, using your understanding of logs.
Re-write each of your answers as a single logarithm, as shown in the example.

Example: $\log_4(16)+\log_4(4)=2+1=3$ This answer, 3, is equal to $\log_4(64)$ $=3$ $\log_4 4^3$

We've shown that: $\log_4(16)+\log_4(4)=\log_4(64)$

1) $\log_2(8)+\log_2(4)=$ _____ $\log_2(\quad)$

2) $\log_3(9)+\log_3(81)=$ _____ $\log_3(\quad)$

these can be written as...

3) $\log_3(\frac{1}{9})+\log_3(81)=$ _____ $\log_3(\quad)$

4) $\log_5(5)+\log_5(1)=$ _____ $\log_5(\quad)$

5) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X+\log_c Y=\log_c(\quad)}$$

Below are some questions that we can't answer directly with the definition of logarithms. Use the pattern discovered above to write each one as a single logarithm, then evaluate it.

6) $\log_6 12+\log_6 3$

7) $\log 250+\log 40$

8) $\log_8(\frac{3}{64})+\log_8(\frac{1}{3})$

Logarithms – Investigation

Part I:

Evaluate the expressions on the left, using your understanding of logs.
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Example: $\log_4(16)+\log_4(4)=2+1=3$ This answer, 3, is equal to $\log_4(64)$

We've shown that: $\log_4(16)+\log_4(4)=\log_4(64)$

1) $\log_2(8)+\log_2(4)=\underline{5}$ $\log_2(32)=5$

$3+2$

2) $\log_3(9)+\log_3(81)=\underline{6}$ $\log_3(729)=6$

$2+4$

these can be written as...

3) $\log_3(\frac{1}{9})+\log_3(81)=\underline{2}$ $\log_3(9)=2$

$-2+4$

4) $\log_5(5)+\log_5(1)=\underline{1}$ $\log_5(5)=1$

$1+0$

5) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X+\log_c Y=\log_c(XY)}$$

Below are some questions that we can't answer directly with the definition of logarithms. Use the pattern discovered above to write each one as a single logarithm, then evaluate it.

6) $\log_6 12+\log_6 3 = \log_6(12\cdot 3)$
$= \log_6(36) = 2$

7) $\log 250+\log 40 = \log(250\cdot 40)$
$= \log(10000)$
$= 4$

8) $\log_8(\frac{3}{64})+\log_8(\frac{1}{3}) = \log_8\left(\frac{\cancel{3}}{64}\cdot\frac{1}{\cancel{3}}\right)$
$= \log_8\left(\frac{1}{64}\right)$
$= \log_8\left(\frac{1}{8^2}\right)$
$= \log_8(8^{-2}) = -2$

This result links to an exponent law we already know:

$\log_c X = a$ means $c^a = x$
$\log_c Y = b$ means $c^b = y$ } definition of logarithm

substituting from above

$\log_c(XY) = \log_c(c^a c^b)$
$= \log_c(c^{a+b})$ using exponent law here
$= a+b$ } definition of logarithm
$= \log_c X + \log_c Y$

Part II:
Evaluate the expressions on the left, using your understanding of logs.
Re-write each of your answers as a single logarithm, as shown in the example.

Example: $\log_4(64) - \log_4(16) = 3 - 2 = 1$ This answer, 1, is equal to $\log_4(4)$

We've shown that: $\log_4(64) - \log_4(16) = \log_4(4)$

9) $\log_5 625 - \log_5 5 =$ ______ $\log_5(\quad)$

10) $\log_6 36 - \log_6 6 =$ ______ $\log_6(\quad)$

which is the same as...

11) $\log_3 9 - \log_3 1 =$ ______ $\log_3(\quad)$

12) $\log_2 16 - \log_2 32 =$ ______ $\log_2(\quad)$

13) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X - \log_c Y = \log_c(\qquad)}$$

Below are more questions that we can't answer directly with the definition of logarithms. Use the new pattern discovered above to write each one as a single logarithm, then evaluate it.

14) $\log_6 72 - \log_6 2$

15) $\log 12 - \log 0.12$

16) $\log_{12} 2 - \log_{12} 288$

Part II:
Evaluate the expressions on the left, using your understanding of logs.
Re-write each of your answers as a single logarithm, as shown in the example.

Example: $\log_4(64) - \log_4(16) = 3 - 2 = 1$ This answer, 1, is equal to $\log_4(4)$

We've shown that: $\log_4(64) - \log_4(16) = \log_4(4)$

9) $\log_5 625 - \log_5 5 = \underline{3}$ $\quad 4 - 1$ $\qquad \log_5(125) = 3$

10) $\log_6 36 - \log_6 6 = \underline{1}$ $\quad 2 - 1$ $\qquad \log_6(6) = 1$

which is the same as...

11) $\log_3 9 - \log_3 1 = \underline{2}$ $\quad 2 - 0$ $\qquad \log_3(9) = 2$

12) $\log_2 16 - \log_2 32 = \underline{-1}$ $\quad 4 - 5$ $\qquad \log_2\left(\frac{1}{2}\right) = -1$

13) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X - \log_c Y = \log_c\left(\frac{X}{Y}\right)}$$

Below are more questions that we can't answer directly with the definition of logarithms. Use the new pattern discovered above to write each one as a single logarithm, then evaluate it.

14) $\log_6 72 - \log_6 2 = \log_6\left(\frac{72}{2}\right)$
$= \log_6(36) = 2$

15) $\log 12 - \log 0.12 = \log\left(\frac{12}{0{,}12}\right)$
$= \log(100) = 2$

16) $\log_{12} 2 - \log_{12} 288 = \log_{12}\left(\frac{2}{288}\right)$
$= \log_{12}\left(\frac{1}{144}\right)$
$= \log_{12}\left(\frac{1}{12^2}\right)$
$= \log_{12}(12^{-2}) = -2$

This result also links to an exponent law we already know:

$\log_c X = a$ means $c^a = x$
$\log_c Y = b$ means $c^b = y$ } definition of logarithm

substituting from above

$\log_c\left(\frac{X}{Y}\right) = \log_c\left(\frac{c^a}{c^b}\right)$
$= \log_c(c^{a-b})$ using exponent law here
$= a - b$ } definition of logarithm
$= \log_c X - \log_c Y$

Your Turn

Use the laws of logarithms to simplify and evaluate each expression.

b) $\log_5 1000 - \log_5 4 - \log_5 2$

$= \log_5\left(\frac{1000}{4}\right) - \log_5 2$
$= \log_5(250) - \log_5 2$
$= \log_5\left(\frac{250}{2}\right)$
$= \log_5 125 = 3$

c) $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

$= \log_3 6^2 - \log_3 64^{1/2} + \log_3 2$
$= \log_3 36 - \log_3 8 + \log_3 2$
$= \log_3\left(\frac{36}{8}\right) + \log_3 2$
$= \log_3\left(\frac{36 \times 2}{8}\right)$

$$= \log_3\left(\frac{72}{8}\right) = \log_3 9 = 2$$

8.3: Laws of Logarithms

Rule of Logarithms

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Rule Name	Property
Log of 1	$\log_b 1 = 0$
Log of the same number as base	$\log_b b = 1$
Product Rule	$\log_b(mn) = \log_b m + \log_b n$
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Rule	$\log_b m^n = n \log_b m$
Change of Base Rule	$\log_a b = \frac{\log_c b}{\log_c a}$ (OR) $\log_a b \cdot \log_c a = \log_c b$
Equality Rule	$\log_b a = \log_b c \Rightarrow a = c$
Number Raised to Log	$b^{\log_b x} = x$
Other Rules	$\log_{b^n} a^m = \frac{m}{n} \log_b a$ $-\log_b a = \log_b \frac{1}{a}$ (OR) $= \log_{\frac{1}{b}} a$

Rule 1: $\log_b(M \cdot N) = \log_b M + \log_b N$

Rule 2: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Rule 3: $\log_b\left(M^k\right) = k \cdot \log_b M$

Rule 4: $\log_b(1) = 0$

Rule 5: $\log_b(b) = 1$

Rule 6: $\log_b\left(b^k\right) = k$

Rule 7: $b^{\log_b(k)} = k$

Where:

$b > 0$ but $b \neq 1$, and M, N, and k are real numbers but M and N must be positive!

- **The Law of Logarithms for Powers (Power Law)** = $\log_a x^n = n \log_a x$
- **The Law of Logarithms for Roots** = $\log_x \sqrt[n]{x^m} = \log_a x^{\frac{m}{n}} = \frac{m}{n} \log_a x$
- **The Multiplication Law of Logs (Product Law)**= $\log_a xy = \log_a x + \log_a y$
- **The Division Law of Logs (Quotient Law)**= $\log_a \frac{x}{y} = \log_a x - \log_a y$

$$
\begin{aligned}
5\log_3(x)+2\log_3(4x)-\log_3\left(8x^5\right) &= \log_3\left(x^5\right)+\log_3\left((4x)^2\right)-\log_3\left(8x^5\right)\\
&= \log_3\left(x^5\right)+\log_3\left(16x^2\right)-\log_3\left(8x^5\right)\\
&= \log_3\left(x^5\cdot 16x^2\right)-\log_3\left(8x^5\right)\\
&= \log_3\left(16x^7\right)-\log_3\left(8x^5\right)\\
&= \log_3\left(\frac{16x^7}{8x^5}\right)
\end{aligned}
$$

$$5\log_3(x)+2\log_3(4x)-\log_3\left(8x^5\right)=\log_3\left(2x^2\right)$$

$$
\begin{aligned}
\log_6\left(\frac{36m^3}{\sqrt{n}}\right) &= \log_6\left(36m^3\right)-\log_6\left(\sqrt{n}\right)\\
&= \log_6(36)+\log_6\left(m^3\right)-\log_6\left(n^{\frac{1}{2}}\right)\\
&= \log_6\left(6^2\right)+3\log_6(m)-\frac{1}{2}\log_6(n)\\
&= 2\log_6(6)+3\log_6(m)-\frac{1}{2}\log_6(n) \quad \text{Rule 5} \Rightarrow \log_6(6)=1\\
&= 2(1)+3\log_6(m)-\frac{1}{2}\log_6(n)
\end{aligned}
$$

$$\log_6\left(\frac{36m^3}{\sqrt{n}}\right)=2+3\log_6(m)-\frac{1}{2}\log_6(n)$$

$$
\begin{aligned}
2\log_5(m)+3\log_5(k)-8\log_5(y) &= \log_5\left(m^2\right)+\log_5\left(k^3\right)-\log_5\left(y^8\right)\\
&= \log_5\left(m^2\cdot k^3\right)-\log_5\left(y^8\right)\\
&= \log_5\left(m^2k^3\right)-\log_5\left(y^8\right)
\end{aligned}
$$

$$2\log_5(m)+3\log_5(k)-8\log_5(y)=\log_5\left(\frac{m^2k^3}{y^8}\right)$$

$$3+\frac{1}{2}\log_4(x)+\frac{1}{2}\log_4(y)=3+\log_4\left(x^{\frac{1}{2}}\right)+\log_4\left(y^{\frac{1}{2}}\right)$$

$$=3+\log_4\left(x^{\frac{1}{2}}\cdot y^{\frac{1}{2}}\right)$$

$$=3+\log_4\left(\sqrt{x}\cdot\sqrt{y}\right)$$

$$=3+\log_4\left(\sqrt{xy}\right)$$

$$=3\cdot\log_4(4)+\log_4\left(\sqrt{xy}\right) \quad \text{Since } \log_4(4)=1$$

$$=\log_4\left(4^3\right)+\log_4\left(\sqrt{xy}\right)$$

$$=\log_4\left(4^3\cdot\sqrt{xy}\right)$$

$$3+\frac{1}{2}\log_4(x)+\frac{1}{2}\log_4(y)=\log_4\left(64\sqrt{xy}\right)$$

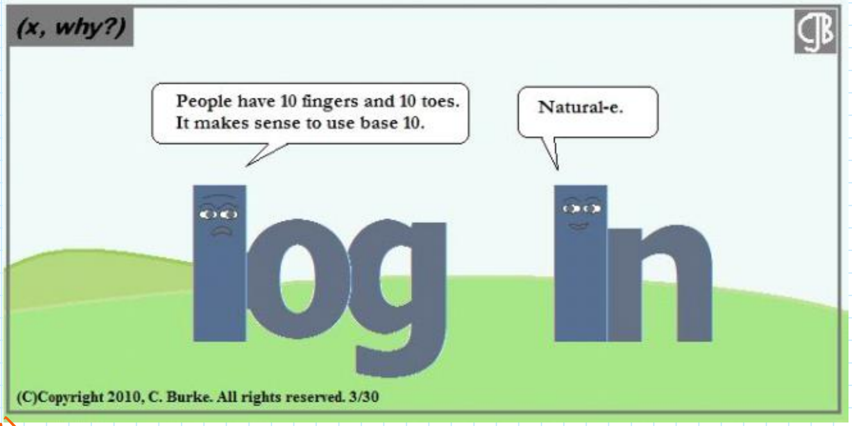
- **Common Base Law =** $\log_a a^x = x$ **OR** $a^{\log_a x} = x$

$\log_a N = x$

$a^x = N$

$a^{\log_a N} = N$

$a^x = N$	$\log_a N = x$
$2^3 = 8$	$\log_2 8 = 3$
$3^4 = 81$	$\log_3 81 = 4$
$5^3 = 125$	$\log_5 125 = 3$
$10^4 = 10000$	$\log_{10} 10000 = 4$
$7^1 = 7$	$\log_7 7 = 1$
$5^0 = 1$	$\log_5 1 = 0$



$$4\left(\log_{36}\sqrt{36}\right) \rightarrow 4\left(\cancel{\log_{36} 36}^{\frac{1}{2}}\right) \rightarrow 4\left(\tfrac{1}{2}\right)=\boxed{2}$$

8.3 Laws of Logarithms

Product Law:	$\log_c(MN) = \log_c M + \log_c N$
Quotient Law:	$\log_c\left(\dfrac{M}{N}\right) = \log_c M - \log_c N$
Power Law:	$\log_c\left(M^P\right) = P\log_c M$

To Try:

1. Evaluate without using use the "log" button: $\log_3 54 - \log_3 2 = \log_3\left(\frac{54}{2}\right)$

subtraction → ÷ quotient law.

$= \log_3 27 \rightarrow \cancel{\log_3} \cancel{3}^3$

$= \boxed{3}$

Recall: $\log_e x = \ln x$

2. Find the value of each of the following without using a calculator:

a) $\ln 1 = 0$ b/c $\cancel{\ln} e^0 = 0$ or $\cancel{\log_e} e^0 = 0$

b) $\ln e = 1$ b/c $\cancel{\log_e} e^1 = 1$

c) $\ln e^4 = 4$ b/c $\cancel{\log_e} e^4 = 4$

3. Evaluate without using the "log" button: $\log_{14} 4 + \log_{14} 49 =$

adding → multiply

$\log_{14}(4 \times 49) = \log_{14} 196$

$= \cancel{\log_{14}} \cancel{14}^2$

$= \boxed{2}$

Change of Base Formula:	$\log_C A = ? \ \dfrac{\log A}{\log C}$

common logs ($\log_{10}$) ∴ use calc.

1. Evaluate. Give answer correct to 4 decimal places. $\log_2 18 = \dfrac{\log 18}{\log 2}$

$\doteq \boxed{4.1699}$

2. Express as a single logarithm. $\dfrac{\log 30}{\log 5} = \boxed{\log_5 30}$

argument

base

3. Rewrite this equation so you can graph it on a graphing calculator: $y = \log_4 x$

$y = \dfrac{\log x}{\log 4}$

Your Turn p.395

Write each expression in terms of individual logarithms of x, y, and z.

a) $\log_6 \dfrac{x}{y}$ ÷ $= \log_6 x - \log_6 y$ quotient law

b) $\log_5 \sqrt{xy}$ root to $\frac{1}{2}$ $= \log_5 (xy)^{\frac{1}{2}}$ ① power law $\rightarrow = \frac{1}{2}\log_5 xy$

② product law $\rightarrow = \frac{1}{2}(\log_5 x + \log_5 y)$

or

$\frac{1}{2}\log_5 x + \frac{1}{2}\log_5 y$

c) $\log_3 \dfrac{9}{\sqrt[3]{x^2}}$ ÷ → −

① product law → $\log_3 9 - \log_3 x^{\frac{2}{3}}$

② power law + common base $\cancel{\log_3} 3^2 - \frac{2}{3}\log_3 x$

$\boxed{2 - \frac{2}{3}\log_3 x}$

d) $\log_7 \dfrac{x^5 y}{\sqrt{z}}$ x → + ÷ → −

① product + quotient

$\log_7 x^5 + \log_7 y - \log_7 z^{\frac{1}{2}}$

d) $\log_7 \frac{x^5 y}{\sqrt{z}}$ ÷ → −

② power law + common base $\log_3 3^2 - \frac{2}{3}\log_3 x$ $\boxed{2-\frac{2}{3}\log_3 x}$

① product + quotient laws $= \log_7 x^5 + \log_7 y - \log_7 z^{\frac{1}{2}}$

② power law $= \boxed{5\log_7 x + \log_7 y - \frac{1}{2}\log_7 z}$

Your Turn p. 396

Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_3 9\sqrt{3}$ → ① common base 3 : $\log_3 3^2 \cdot 3^{\frac{1}{2}} \rightarrow \cancel{\log_3} \cancel{3}^{\frac{5}{2}} = \boxed{\frac{5}{2}}$

b) $\log_5 1000 - \log_5 4 - \log_5 2$

c) $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

① quotient law $\log_5\left(\frac{1000}{4} \div 2\right) = \log_5\left(\frac{1000}{4} \times \frac{1}{2}\right)$ multiply by reciprocal!

② simplify + common base $= \log_5\left(\frac{1000}{8}\right)$

$= \log_5 125$

$= \cancel{\log_5} \cancel{5}^3$

$= \boxed{3}$

① power law $\log_3 6^2 - \log_3 64^{\frac{1}{2}} + \log_3 2$

$\log_3 36 - \log_3 8 + \log_3 2$

② quotient + product $\log_3\left(\frac{36}{8} \times 2\right)$ NOT $\log_3\left(\frac{36}{8 \times 2}\right)$ wrong.

$= \log_3 9$

③ common base $= \cancel{\log_3} \cancel{3}^2$

$= \boxed{2}$

1. $\log_3(4y^2)$ $= \boxed{\log_3 4 + 2\log y}$	6. $\frac{1}{2}(\log b - \log c)$
2. $2\log_4 b + 3\log_4 c$ $\log_4 b^2 + \log_4 c^3$ $= \boxed{\log_4 b^2c^3}$	7. $\log\left(\frac{\sqrt{a}}{c^2}\right)$
3. $\ln(ab)$	8. $\log(x^2y)^4$
4. $\log\left(\frac{a}{b}\right)$	9. $3\log x - \log w^2$
5. $\frac{1}{2}\log a + 2\log c$	10. $\log\left(\frac{1000a^2}{c}\right)$ $= \log 1000 + \log a^2 - \log c$ $= \log 10^3 + \log a^2 - \log c$ $= \boxed{3 + 2\log a - \log c}$

11. $\log_5\left(5x\sqrt{y}\right)$	16. $\log_7 y - 2\log_7 w + \log_7(5x)$
12. $\log\left(\dfrac{\sqrt{bc}}{a}\right)$	17. $\log a + 3\log b - 2\log c$
13. $2\log a - 4\log b$	18. $5\log_4 2 - \dfrac{1}{3}\log_4 8$
14. $\log\left(a^2c\right)$	19. $\dfrac{\log_5 x}{4} - \log_5(3x)$
15. $\log\left(\dfrac{x}{yw}\right)$	20. $2\log c - (3\log a + \log b)$