Class_18 Mar 9 Sine and Cosine Laws

Monday, March 6, 2023 11:06 AM

Tonight's Class:

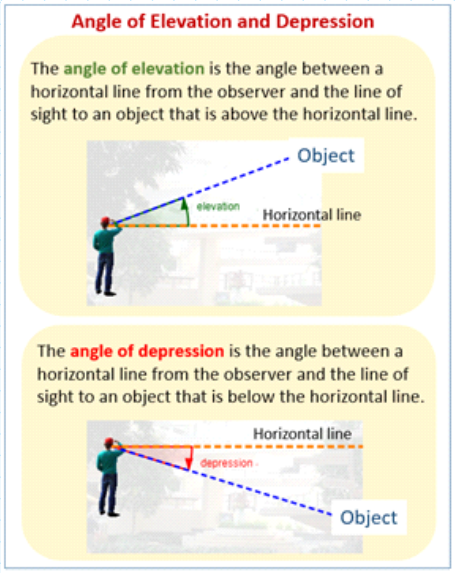
- **Working through sections 5.5, 5.6**
  - **Sine Law (continued)**
  - **Cosine Law**
- **Work on practice questions from worktext**

To try

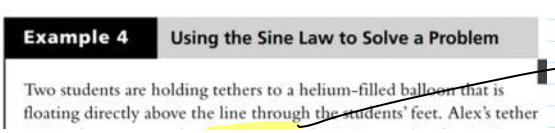
Triangle ABC has these measurements: A = 38 degrees, a = 15 cm, b = 23 cm.
Solve this triangle, correct to 1 decimal place.

✓

Always check if there is more than one △.



WT p 473

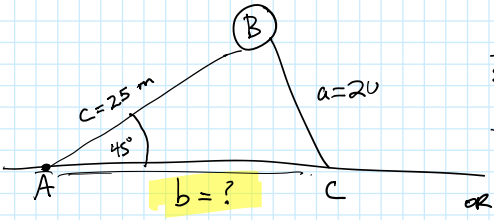


→ inclination / elevation
declination / depression

Example 4 **Using the Sine Law to Solve a Problem**

Two students are holding tethers to a helium-filled balloon that is floating directly above the line through the students' feet. Alex's tether is 25 m long at an angle of inclination of 45°. Carmen's tether is 20 m long. To the nearest metre, determine the distance between Alex and Carmen.

→ inclination / elevation
declination / depression



$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{25}{\sin C} = \frac{20}{\sin 45^\circ}$$

or $\frac{\sin C}{25} = \frac{\sin 45^\circ}{20}$

$$\sin C = \frac{25 \sin 45^\circ}{20}$$

$$C = \sin^{-1}\left(\frac{25 \sin 45^\circ}{20}\right)$$

$$C = 62.114\ldots^\circ$$

Is there another possible Δ?

supplement

$$C = 180^\circ - 62.11\ldots^\circ$$

$$C = 117.885\ldots^\circ$$

$$B = 180^\circ - 45^\circ - 62.11\ldots^\circ$$

$$B = 72.8855\ldots^\circ$$

$$B = 180^\circ - 45^\circ - 117.885\ldots^\circ$$

$$B = 17.1144332^\circ$$

★ Keep all the info, don't round now.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin(72.88\ldots^\circ)} = \frac{20}{\sin 45^\circ}$$

$$b = \frac{20 \sin 72.88\ldots^\circ}{\sin 45^\circ}$$

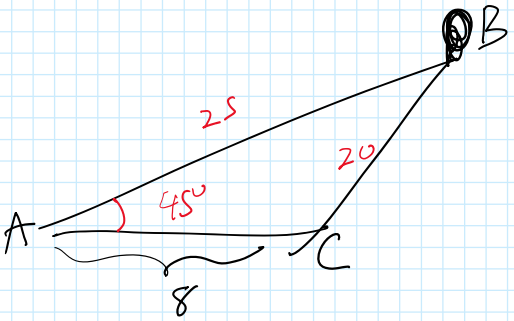
$$\frac{b}{\sin(17.1144332^\circ)} = \frac{20}{\sin 45^\circ}$$

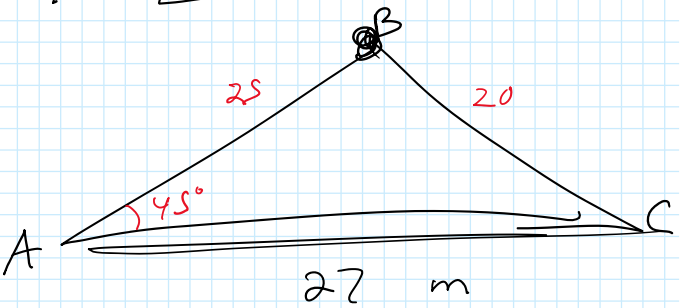
$$b = \frac{20 \sin 17.114\ldots^\circ}{\sin 45^\circ}$$

$b \doteq 27.0318$

$\Rightarrow$ **27 m**

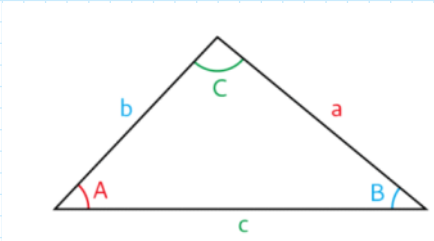
b ≐ 8 m





5.6 Cosine Law

Focus: Apply the Cosine Law to solve problems in triangles that are not right triangles.



Law of Cosines

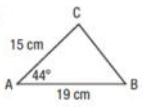
Standard Form	*Alternative Form*
$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

WT, page 490

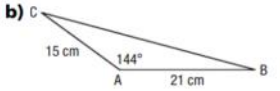
Example 1 **Using the Cosine Law to Determine the Length of a Side**

In each triangle, determine the length of BC to the nearest tenth of a centimetre.

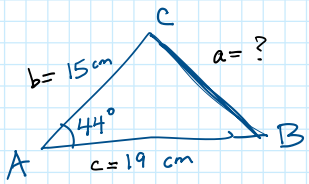
a)



b)



a)



$a^2 = b^2 + c^2 - 2bc\cos A$

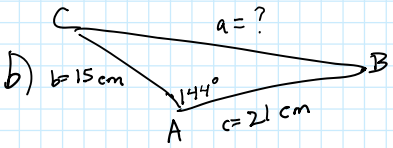
$a^2 = (15)^2 + (19)^2 - 2(15)(19)\cos 44^\circ$

$\sqrt{a^2} = \sqrt{175.976\,3138}$

$a = 13.265\,606\,42$

$a \doteq 13.3$ cm

b)



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 15^2 + 21^2 - 2(15)(21)\cos 144^\circ$

$\sqrt{a^2} = \sqrt{1175.680\,706}$

$a = 34.288\,200\,69$

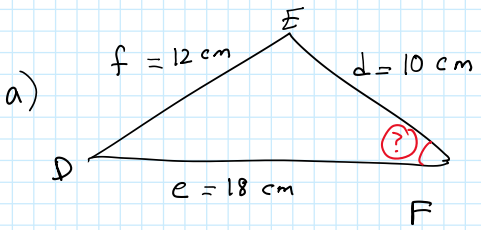
$a \doteq 34.3$ cm

WT, page 491

Example 2 **Using the Cosine Law to Determine the Measure of an Angle**

a) In ΔDEF, DF = 18 cm, DE = 12 cm, and EF = 10 cm; determine the measure of ∠F to the nearest degree.
b) In ΔGHJ, GJ = 18 cm, GH = 21 cm, and HJ = 10 cm; determine the measure of ∠J to the nearest degree.

a)



Alternative Form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

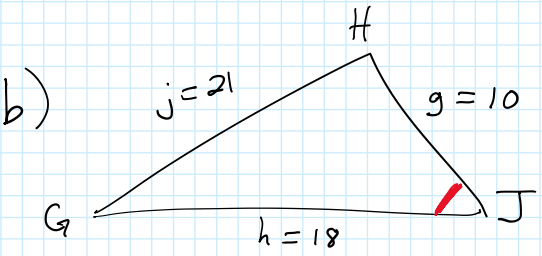
$$\cos F = \frac{d^2 + e^2 - f^2}{2de}$$

$$\cos F = \frac{10^2 + 18^2 - 12^2}{2(10)(18)}$$

$$F = \cos^{-1}\left(\frac{10^2 + 18^2 - 12^2}{2(10)(18)}\right)$$

$$F \doteq 39^\circ$$

b)



$$\cos J = \frac{g^2 + h^2 - j^2}{2(g)(h)}$$

$$\cos J = \frac{10^2 + 18^2 - 21^2}{2(10)(18)}$$

$$J = \cos^{-1}\left(\frac{10^2 + 18^2 - 21^2}{2(10)(18)}\right)$$

$$J \doteq 93^\circ$$

If instead they ask us to SOLVE the triangle, using cosine law:

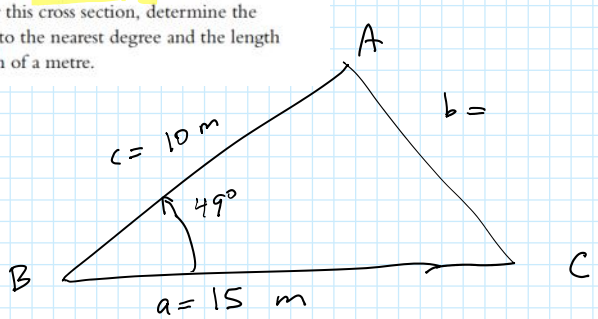
1) **It's best to find the LARGEST angle first**
2) **You can then use Sine Law to get the second angle**

Doing it this way avoids running into ambiguous case

WT, page 492

Example 3 **Using the Cosine Law and the Sine Law to Solve a Problem**

A 10 m by 20 m rectangular array of solar panels is to be installed on the flat roof of a factory in Lethbridge, Alberta. The angle of inclination for the array should match the latitude of Lethbridge, which is 49°. The 20-m side of the array rests on the roof. The cross section of the support, the roof, and the panels form a triangle with a base that is 15 m long. For this cross section, determine the measures of the other two angles to the nearest degree and the length of the support to the nearest tenth of a metre.



$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 15^2 + 10^2 - 2(15)(10)\cos 49°$

$\sqrt{b^2} = \sqrt{128.1822913}$

$b = 11.32176185$

rounds to 11.3 m

★ We need to use the full "version" of b, not the rounded value, as we keep on solving for the angles

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin A}{15} = \frac{\sin 49°}{11.3217\ldots}$

$\sin A = \frac{15\sin 49°}{11.3217\ldots}$

don't use 11.3!

$A = \sin^{-1}\left(\frac{15\sin 49°}{11.3217\ldots}\right)$

$A = 89.19474472°$, $A \doteq 89°$

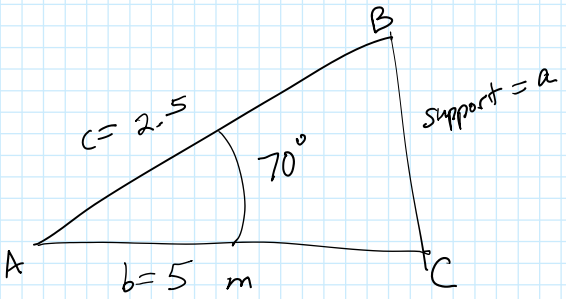
$C = 180° - 89° - 49°$

$C = 42°$

CYU, page 492

Check Your Understanding

3. A retaining wall is leaning at an angle of 70° to the horizontal. A rigid support is to be placed 5.0 m from the base of the wall and it will be attached to the wall 2.5 m from its base. Determine the length of the support to the nearest tenth of a metre and the measure of the angle between the support and the wall to the nearest degree.



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 5^2 + 2.5^2 - 2(5)(2.5)\cos 70°$

$a \doteq 4.8$ m but actually 4.764398852

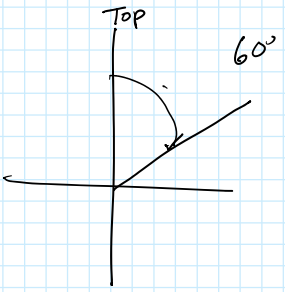
$\frac{\sin B}{5} = \frac{\sin 70°}{4.764398852}$, $B = \sin^{-1}\left(\frac{5\sin 70°}{4.76\ldots}\right)$

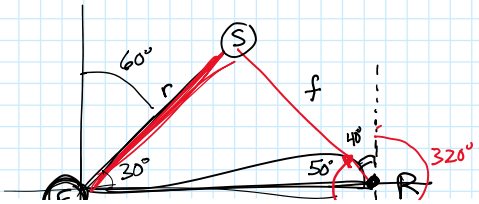
$B \doteq 80°$

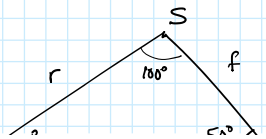
WT, page 499

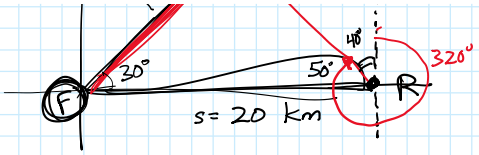
9. A fire spotter sees smoke on a bearing of 060°. At a point 20 km due east of the fire spotter, a ranger sees the same smoke on a bearing of 320°.

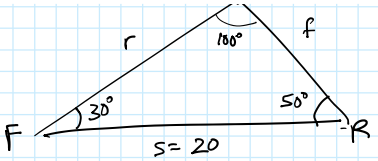
a) How far is the smoke from each location?











$$\frac{f}{\sin F} = \frac{s}{\sin S}$$

$$\frac{f}{\sin 30^\circ} = \frac{20}{\sin 100^\circ}$$

$$f = \frac{20 \sin 30^\circ}{\sin 100^\circ}$$

$$f = 10.15426612$$

$$f \doteq 10 \text{ km}$$

$$\frac{r}{\sin R} = \frac{s}{\sin S}$$

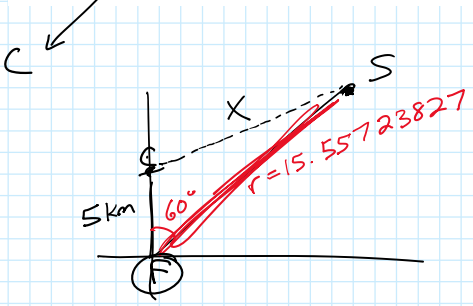
$$\frac{r}{\sin 50^\circ} = \frac{20}{\sin 100^\circ}$$

$$r = \frac{20 \sin 50^\circ}{\sin 100^\circ}$$

$$r = 15.55723827$$

$$r \doteq 16 \text{ km}$$

b) A fire crew is 5 km due north of the fire spotter. How far is the crew from the smoke? Give the answers to the nearest kilometre.



$$x^2 = 5^2 + (15.55\ldots)^2 - 2(5)(15.55\ldots)\cos 60^\circ$$

$$\sqrt{x^2} = \sqrt{189.2414712}$$

$$x = 13.75650651$$

$$x \doteq 14 \text{ km}$$

OMIT p 493 example #4
p 502, #13

Coming up

- **Spring break - our next class will be on Tuesday, March 28**
- **Work on the worktext questions for chapter 5, except for section 5.4**
- **Complete the Chapter 5 hand-in assignment. Due March 28.**
- **Prepare for the Chapter 5 Test, on Thursday, March 30.**

(Also, there are extra Cosine + Sine Law worksheets on ...)