Class_18 Nov 8 - Exp Equations and Logs

Sunday, November 6, 2022 8:44 PM

Tonight's Class:

- **7.3 Solving Exponential Equations**
- **8.1 Understanding Logarithms**
- **Chapter 7 Review (test next class)**

In 2022, the population of Abbotsford, BC, was about 168,000. Its annual growth rate was 2.2%.

$100\% + 2.2\% = 102.2\%$

Create an equation that describes this situation and use it to estimate what the population will be in 2030 if the growth rate stays the same. (Round down, giving the answer correct to the nearest whole person.)

$$\begin{array}{r} 2030 \\ -2022 \\ \hline 8 \end{array}$$

$$P = 168\,000(1.022)^8$$

$$= 199\,947.7\ldots$$

$$\boxed{199\,947 \text{ people}}$$

Do you remember the exponent laws?
What if you don't?
Can you figure them out?

7.3 Solving Exponential Equations

Remember the rules for working with exponents:

$a^m a^n = a^{m+n}$

example, and a way to figure it out

$x^2 \cdot x^5$
$= (x \cdot x)(x \cdot x \cdot x \cdot x \cdot x)$
$= x^7$

$(a^m)^n = a^{mn}$

$(ab)^m = a^m b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$\frac{a^m}{a^n} = a^{m-n}$

$\frac{x^6}{x^2} = \frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}} = x^4$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$5^{1/2} = \sqrt[2]{5^1}$

$\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5$

$5^{1/2} \times 5^{1/2} = 5^{1/2 + 1/2} = 5^1$

∴ We can see that $\sqrt{5}$ and $5^{1/2}$ are the same thing.

Exponential equations are ones where the ***variable is in the exponent***. We can solve these equations by

- Writing the left side of the equation and the right side of the equation so they each use the same base.
- Then, we use the fact that if $\boxed{a^x = a^y}$ it forces $x = y$, to finish solving the equation.

1) make both sides of the equation use the same base
2) set the exponents equal

Example

$$8^{4x-1} = \left(\frac{1}{2}\right)^{x+5}$$

$$(2^3)^{4x-1} = (2^{-1})^{x+5}$$

$$2^{12x-3} = 2^{-x-5}$$

$$\Rightarrow 12x-3 = -x-5$$

$$12x - 3 = -x - 5$$
$$+x \qquad +x$$
$$13x - 3 = -5$$
$$+3 \qquad +3$$
$$\frac{13x}{13} = \frac{-2}{13}$$
$$\boxed{x = -\frac{2}{13}}$$

To Try

1. Rewrite the expressions so they have the same base, then solve the equation.

a) $\left(\frac{1}{25}\right)^{4x} = (125)^{3x+2}$

$$(25^{-1})^{4x} = (125)^{3x+2}$$
$$[(5^2)^{-1}]^{4x} = (5^3)^{3x+2}$$
$$5^{-8x} = 5^{9x+6}$$
$$\Rightarrow -8x = 9x+6$$

$$-8x = 9x + 6$$
$$-9x \qquad -9x$$
$$\frac{-17x}{-17} = \frac{6}{-17}$$
$$\boxed{x = \frac{6}{-17}}$$

b) $(8)^{2x+7} = 16^{4x+2}$

$$(2^3)^{2x+7} = (2^4)^{4x+2}$$
$$2^{6x+21} = 2^{16x+8}$$

$$6x + 21 = 16x + 8$$
$$-16x \qquad -16x$$
$$-10x + 21 = 8$$
$$-21 \qquad -21$$
$$\frac{-10x}{-10} = \frac{-13}{-10}$$
$$\boxed{x = \frac{13}{10}}$$

c) $16^{3x} = 8^{3x-1}64^{x}$

$$(2^4)^{3x} = (2^3)^{3x-1}(2^6)^{x}$$
$$2^{12x} = \underbrace{2^{9x-3}\,2^{6x}}$$
$$2^{12x} = 2^{9x-3+6x}$$
$$2^{12x} = 2^{15x-3}$$
$$\Rightarrow 12x = 15x - 3$$
$$-15x \qquad -15x$$

$$\frac{-3x}{-3} = \frac{-3}{-3}$$
$$\boxed{x = 1}$$

2. For how long does one need to invest \$2000 in an account that earns 6.1% compounded quarterly, before it increases in value to \$2500? Round answer to the nearest quarter of a year.

0.061

$$\frac{0.061}{4}$$

$$A = P(1+i)^n$$

P = principal amount deposited

i = interest rate per compounding period, in decimal form

n = number of compounding periods

$$\frac{2500}{2000} = \frac{\cancel{2000}}{\cancel{2000}}\left(1 + \frac{0.061}{4}\right)^n$$

To solve either

$x = 14.744 \doteq 15$ compounding

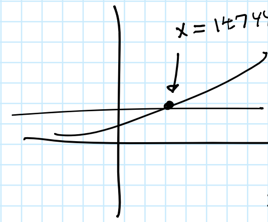
$$\frac{2500}{2000} = \frac{2000}{2000}\left(1 + \frac{0.061}{4}\right)$$

$$1.25 = (1.01525)^n$$

$$\boxed{\begin{array}{l} Y_1 = 1.25 \\ Y_2 = 1.01525 \wedge x \end{array}}$$

$X = 14.744$

$\boxed{\text{To solve either 1) guess \& check OR 2) solve graphically}}$



$x = 14.744 \doteq 15$ compounding periods

$\frac{15}{4} = 3.75$ years

3. The population of a town triples every 6 years. If 4000 people lived there in 2009, how many will be in the town 2030? (Round down to the nearest whole person.)

$$A = 4000(3)^{\frac{21}{6}}$$

$2030 - 2009 = 21$ yrs

$= 187\,061$ people

(7.3) TB p 364: 1, 2, 3ac, 4, 5ac, 7aceg, 9-13

Chapter 7 in-class practice - work in groups

Chapter 8

In a group:

Let's start with a SUPER FUN puzzle!

Take a guess at what these statements are saying:

$power_2(8) = 3$
$power_2(32) = 5$
$power_3(9) = 2$
$power_3(81) = 4$
$power_5(25) = 2$

$2^3 = 8$

Now see if you can fill in the blanks:

$power_2(16) = \underline{4}$

$power_6(36) = \underline{2}$

$power_{5}(\underline{125}) = 3$

5^3

$power_{2}(\underline{\tfrac{1}{2}}) = -1$

$2^{-1} = \frac{1}{2}$

$power_{10}\left(\frac{1}{1000}\right) = \underline{-3}$

$power_{\underline{7}}(49) = 2$

$power_{\underline{5}}(625) = 4$

$power_{16}(\underline{64}) = \frac{3}{2}$

$10^{?} = \frac{1}{1000}$

$10^{?} = \frac{1}{10^3}$

$\square^2 = 49$

$16^{3/2} = \sqrt[2]{16^3}$

$= (\sqrt[2]{16})^3$

$= (4)^3$

$= 64$

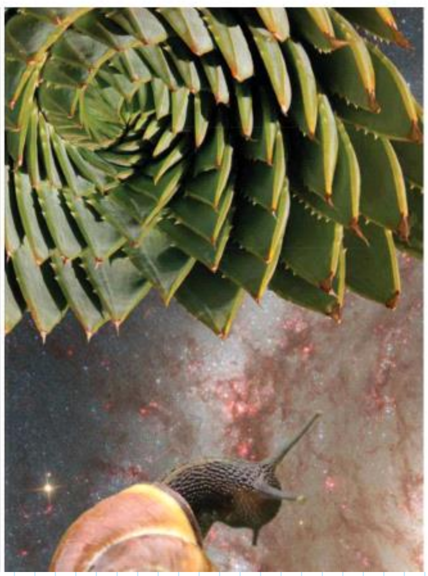
TB p 370

CHAPTER 8

Logarithmic Functions

Logarithms were developed over 400 years ago, and they still have numerous applications in the modern world. Logarithms allow you to solve any exponential equation. Logarithmic scales use manageable numbers to represent quantities in science that vary over vast ranges, such as the energy of an earthquake or the pH of a solution. Logarithmic spirals model the spiral arms of a galaxy, the curve of animal horns, the shape of a snail, the growth of certain plants, the arms of a hurricane, and the approach of a hawk to its prey.

In this chapter, you will learn what logarithms are, how to represent them, and how to use them to model situations and solve problems.



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Chapter 8: Logarithmic Functions

8.1 Understanding Logarithms

A logarithm tells how many copies of one number we need to multiply together, to create a different number.

For example:
How many 4's do we have to multiply together to get 64?
$4 \times 4 \times 4 = 64$, which shows we have to multiply 3 of the "4's" to produce 64
This tells us the ***logarithm*** is 3.

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Because $4 \times 4 \times 4 = 64$, we can say: $\log_4(64) = 3$

- we can read this as "The logarithm base 4 of 64 is equal to 3"
- we can shorten it a bit, and say "log base 4 of 64 equals 3"

logarithm is an exponent

$$\log_4 (\underset{\text{argument}}{64}) = 3$$

base

A logarithm tells us how many copies of the BASE we need to multiply together, to create the ARGUMENT – in other words, the logarithm is the ***exponent*** we raise the base to, in order to produce the argument.

I think, "4 to what exponent equals 16?"

Try These

1. $\log_4(16) = 2$
   I know this is right, because: $4^2 = 16$
2. $\log_3(27) = 3$
   $3^3 = 27$
3. $\log_{0.5}(0.5) = 1$
   $0.5^1 = 0.5$
4. $\log_a(a^7) = 7$
5. $\log_2(2^{-3}) = -3$
   (a horse)

6. $\log_1 7 =$ does not exist
   $1^? = 7$
7. $\log_5\left(\frac{1}{25}\right) = \log_5\left(\frac{1}{5^2}\right) = \log_5\left(5^{-2}\right) = -2$
8. $\log_3 0 =$ does not exist
9. $\log_2(-4) =$ does not exist
10. $\log_6\left(\sqrt[2]{6}\right) = \log_6 6^{1/2} = \frac{1}{2}$
11. $\log_2\left(\sqrt[3]{2^4}\right) = \log_2\left(2^{4/3}\right) = \frac{4}{3}$
12. $\log_8(1) = 0$
    (because: $8^0 = 1$)

For a logarithm to make sense, we need the argument and the base to obey these restrictions:

argument > 0 **base** > 0, base $\neq 1$

TB p 374

Evaluate.

a) $\log_2 32 = 5$

b) $\log_9 \sqrt[5]{81} = \log_9 \sqrt[5]{9^2} = \log_9 9^{2/5} = \frac{2}{5}$

c) $\log_{10} 1\,000\,000 = 6$

nothing written in, means it's base 10

d) $\log_3(9\sqrt{3})$

$= \log_3\left(3^2 3^{1/2}\right)$

$= \log_3\left(3^{2\frac{1}{2}}\right)$

$= 2\frac{1}{2}$ or 2.5

or $\frac{5}{2}$

Notation

log base 10 is called **COMMON log** $\log_{10} x$ *is written* $\log x$

log base *e* is called **NATURAL log** $\log_e x$ *is written* $\ln x$

The number *e* is a very important irrational number. Its decimal expansion starts out:
$e \approx \mathbf{2.71828182845904523536028747 13\ldots}$

https://www.popularmechanics.com/science/math/a24383/mathematical-constant-e/

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Changing Form

Exponents and logarithms are closely connected. Look at these two equations:

$\log_4 64 = 3$ **logarithmic form** and $4^3 = 64$ **exponential form**

Both equations show the relationship between the numbers 4, 3 and 64. We need to know how to change equations from one form to the other, as in some questions one form is better than the other.

To Try

1. Change form.

a) $\log_6 216 = 3$

$6^3 = 216$

b) $\log_p q = r$ ✓

$p^r = q$

c) $\log_{10} 1000 = 3$

$10^3 = 1000$

d) $7^2 = 49$

$\log_7 49 = 2$

e) $5^{x+y} = a$

$\log_5 a = x+y$

f) $49^{1/2} = 7$

$\log_{49} 7 = \frac{1}{2}$

2. Solve for *x*.

a) $\log_2(x-1) = 3$

Change form: $2^3 = x-1$

$8 = x-1$

$\boxed{x=9}$

b) $\log_6 x = -2$

$6^{-2} = x$

$x = \frac{1}{6^2} = \frac{1}{36}$

c) $\log_x 8 = 3$

$x^3 = 8$

$x = 2$

d) $\ln_e x = 2$

$e^2 = x$, $x \doteq 7.39$

e) $\log_2(\log_9 x) = -1$

$\log_2(\text{[scribbled]}) = -1$

$2^{-1} = \text{[scribbled]}$

$2^{-1} = \log_9 x$

$\frac{1}{2} = \log_9 x$

OR $\log_9 x = \frac{1}{2}$

$9^{1/2} = x$, $x = \sqrt{9}$

$\boxed{x = 3}$

f) $4^{\log_4 7} = x$

$4^3 = x$

$\log_4 x = 3$

$\log_4 x = \log_4 7$

$\boxed{x = 7}$

TB p 375

Determine the value of x.

a) $\log_4 x = -2$

b) $\log_{16} x = -\frac{1}{4}$

c) $\log_x 9 = \frac{2}{3}$

$4^{-2} = x$

$\frac{1}{4^2} = x$

$x = \frac{1}{16}$

Coming up:

- **NO CLASS on Thursday, Nov 10 - see you on Tuesday, Nov 15!**
- **Tuesday, Nov 15**
  - **Chapter 7 Hand-in due**
  - **Chapter 7 Test, includes one trig question**

Practice

(7.3) TB p 364: 1, 2, 3ac, 4, 5ac, 7aceg, 9-13

(8.1) TB p 380: 1-4, 8, 10, 12-15

Optional worksheet: Chapter 7 Review Worksheet, on website

Short video clip about logarithms:

https://www.youtube.com/watch?v=zzu2POfYv0Y starting at 0:14