Class_19 June 6 - Log Equations and Applications

Tuesday, June 6, 2023 10:35 AM

Tonight's Class:

- **8.3 Logarithm Laws (continued)**
- **8.4 Log Equations and Applications**

Know these 4 log laws.

1) Product Law: $\log(MN) = \log M + \log N$

2) Quotient Law: $\log\left(\frac{M}{N}\right) = \log M - \log N$

3) Power Law: $\log(M^P) = P\log M$

4) Change of Base Law: $\log_C A = \frac{\log_B A}{\log_B C}$

We can use log laws to

- **"expand" an expression with one argument**
- **"condense" an expression with several arguments**

EXPAND

1. $\log_3(4y^2)$

$= \log_3 4 + \log_3 y^2$

$= \log_3 4 + 2\log_3 y$

2. $2\log_4 b + 3\log_4 c$

$= \log_4 b^2 + \log_4 c^3$

$= \log_4(b^2c^3)$

3. $\ln(ab)$

$= \ln a + \ln b$

$= \log_e a + \log_e b$

4. $\log\left(\frac{a}{b}\right)$ $= \log a - \log b$

5. $\frac{1}{2}\log a + 2\log c$

$= \log a^{1/2} + \log c^2$

$= \log\left(a^{1/2}c^2\right)$

$= \log\left(\sqrt{a}\,c^2\right)$

6. $\frac{1}{2}(\log b - \log c)$ $= \frac{1}{2}\left(\log\frac{b}{c}\right)$

$= \log\left(\frac{b}{c}\right)^{1/2}$

$= \log\sqrt{\frac{b}{c}}$

OR

$\frac{1}{2}(\log b - \log c)$

$= \frac{1}{2}\log b - \frac{1}{2}\log c$

$= \log b^{1/2} - \log c^{1/2}$

$= \log\left(\frac{b^{1/2}}{c^{1/2}}\right) = \log\frac{\sqrt{b}}{\sqrt{c}}$

$= \log\sqrt{\frac{b}{c}}$

7. $\log\left(\frac{\sqrt{a}}{c^2}\right)$ $= \log\sqrt{a} - \log c^2$

$= \log a^{1/2} - \log c^2$

$= \frac{1}{2}\log a - 2\log c$

8. $\log(x^2y)^4$ $= \log(x^8y^4)$

$= \log x^8 + \log y^4$

$= 8\log x + 4\log y$

OR

$\log(x^2y)^4$

$= 4\log(x^2y)$

$= 4(\log x^2 + \log y)$

$= 4(2\log x + \log y)$

$= 8\log x + 4\log y$

9. $3\log x - \log w^2$

$= \log x^3 - \log w^2$

$= \log\left(\frac{x^3}{w^2}\right)$

10. $\log\left(\frac{1000a^2}{c}\right)$

$= \log 1000a^2 - \log c$

$= \log_{10} 1000 + \log a^2 - \log c$

$= 3 + 2\log a - \log c$

11. $\log_5(5x\sqrt{y})$ $= \log_5 5 + \log_5 x + \log_5 y^{1/2}$ $= 1 + \log_5 x + \frac{1}{2}\log_5 y$	16. $\log_7 y - 2\log_7 w + \log_7(5x)$ $= \log_7 y - \log_7 w^2 + \log_7(5x)$ $= \log_7\left(\frac{y}{w^2}\right) + \log_7(5x)$ $= \log_7\left(\frac{y}{w^2}\cdot\frac{5x}{1}\right) = \log_7\left(\frac{5xy}{w^2}\right)$
12. $\log\left(\frac{\sqrt{bc}}{a}\right) = \log\sqrt{bc} - \log a$ $= \log(bc)^{1/2} - \log a$ $= \frac{1}{2}(\log b + \log c) - \log a$ $= \frac{1}{2}\log b + \frac{1}{2}\log c - \log a$	17. $\log a + 3\log b - 2\log c$ $= \log a + \log b^3 - \log c^2$ $= \log(ab^3) - \log c^2$ $= \log\left(\frac{ab^3}{c^2}\right)$
13. $2\log a - 4\log b$ $= \log a^2 - \log b^4$ $= \log\left(\frac{a^2}{b^4}\right)$	18. $5\log_4 2 - \frac{1}{3}\log_4 8$ $= \log_4 2^5 - \log_4 8^{1/3}$ $= \log_4\left(\frac{2^5}{8^{1/3}}\right)$ $\log_4\left(\frac{32}{2}\right)$ $= \log_4(16)$ $= 2$
14. $\log(a^2c)$ $= \log a^2 + \log c$ $= 2\log a + \log c$	19. $\frac{\log_5 x}{4} - \log_5(3x)$ $= \frac{1}{4}\log_5 x - \log_5(3x)$ $= \log_5 x^{1/4} - (\log_5 3 + \log_5 x)$ $= \log_5 x^{1/4} - \log_5 3 - \log_5 x$
15. $\log\left(\frac{x}{yw}\right) = \log x - \log(yw)$ $= \log x - [\log y + \log w]$ $= \log x - \log y - \log w$	20. $2\log c - (3\log a + \log b)$ $= \log c^2 - (\log a^3 + \log b)$ $= \log c^2 - \log(a^3b)$ $= \log\left(\frac{c^2}{a^3b}\right)$

now, let's try condensing it totally:

$\frac{1}{4}\log_5 x - \log_5(3x)$

$= \log_5 x^{1/4} - \log_5(3x)$

$= \log_5\left(\frac{x^{1/4}}{3x^1}\right) = \log_5\left(\frac{x^{1/4 - 1\cdot\frac{4}{4}}}{3}\right)$

$= \log_5\left(\frac{x^{1/4 - 4/4}}{3}\right)$

$= \log_5\left(\frac{x^{-3/4}}{3}\right)$

$= \log_5\left(\frac{1}{3x^{3/4}}\right)$

Know these 4 log laws.

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3) Power Law: $\log(M^P) = P\log M$
4) Change of Base Law: $\log_C B = \frac{\log B}{\log C}$

$\log_5 43 \doteq 2.3370$

$\frac{\log 43}{\log 5}$

Small WB - Using Log Laws

TB p 396

Your Turn

Use the laws of logarithms to simplify and evaluate each expression.

b) $\log_5 1000 - \log_5 4 - \log_5 2$

You looked at these in yesterday's class

c) $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

TB p 401

8. If $\log 3 = P$ and $\log 5 = Q$, write an algebraic expression in terms of P and Q for each of the following.

a) $\log \frac{3}{5}$ $= \log 3 - \log 5 = P - Q$

Try these:

b) $\log 15$ $= \log(3 \cdot 5) = \log 3 + \log 5 = P + Q$

c) $\log 3\sqrt{5}$ $= \log 3 + \log \sqrt{5} = \log 3 + \log 5^{1/2} = \log 3 + \frac{1}{2}\log 5$

$= P + \frac{1}{2}Q$

d) $\log \frac{25}{9}$

$\log 25 - \log 9$

$= \log 5^2 - \log 3^2$

$= 2\log 5 - 2\log 3 = 2Q - 2P$

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8.4 Logarithmic and Exponential Equations

Solving Logarithmic Equations

1. Use logarithm laws to simplify equation into one of two forms:
   - **$\log_c$ (argument) = number**
     - in this case, change to exponential form and solve
   - **$\log_c$ (argument) = $\log_c$ (another argument)**
     - in this case, set the two arguments equal

2. Use algebra to solve the equation you created in step 1.

3. Substitute each solution into the original equation. If the solution makes the argument become zero or a negative number, then it is an ***extraneous solution*** and must be rejected.

Example

1) $\log(3x) + \log(2) = -1$

$\log_{10}(6x) = -1$

$\Rightarrow 10^{-1} = 6x$

$\Rightarrow \quad 10 = 6x$

$\frac{1}{6}\left(\frac{1}{10}\right) = (\cancel{6}x)\cdot\frac{1}{\cancel{6}}$

$\frac{1}{60} = x$

Check this answer is okay: $\log(\underbrace{3\cdot\tfrac{1}{60}}_{\text{not 0, not negative}})$

$\Rightarrow$ Keep answer!

Here we are simply checking to make sure the answer is not extraneous. If we substitute an answer into the original equation and it makes ANY of the arguments 0 or negative, we have to reject that answer. They call these rejected answers "extraneous roots."

2) $\log(x+4) + \log(2) = \log(7x)$

$\log((x+4)(2)) = \log(7x)$

$\log(2x+8) = \log(7x)$

$\Rightarrow \quad 2x+8 = 7x$

$\quad -2x \qquad -2x$

$\frac{8}{5} = \frac{\cancel{5}x}{\cancel{5}}$

$\boxed{x = 8/5}$

Since x = 8/5 does not make any of the original arguments 0 or negative, it is not extraneous.

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2. Use algebra to solve the equation you created in step 1.

3. Substitute each solution into the original equation. If the solution makes the argument become zero or a negative number, then it is an ***extraneous solution*** and must be rejected.

To Try:
Solve for x. Reject any extraneous solutions.

1. $\log_9 5 + \log_9 x = \log_9 30$

$\log_9(5x) = \log_9(30)$

$\Rightarrow \frac{5x}{5} = \frac{30}{5}$

$\boxed{x = 6}$

2. $\ln x + \ln 5 = 2$

$\ln(5x) = 2$

or $\log_e(5x) = 2$

$\frac{e^2}{5} = \frac{5x}{5}$ $x = \frac{e^2}{5} \doteq \boxed{1.48}$

3. $\ln 512 - \ln 8 = 3\ln x$

$\ln\left(\frac{512}{8}\right) = \ln(x^3)$

$\ln(64) = \ln(x^3)$

$\Rightarrow (64)^{1/3} = (x^3)^{1/3}$

$\boxed{x = 4}$

4. $\log_2(x-6) = 3 - \log_2(x-4)$

$\log_2(x-6) + \log_2(x-4) = 3$

$\log_2[(x-6)(x-4)] = 3$

$\log_2(x^2 - 4x - 6x + 24) = 3$

$\log_2(x^2 - 10x + 24) = 3$

$2^3 = x^2 - 10x + 24$

$8 = x^2 - 10x + 24$
$-8 \qquad\qquad -8$

$0 = x^2 - 10x + 16$

$0 = (x-2)(x-8)$

$\cancel{x = 2}$ $\boxed{x = 8}$

makes argument negative. REJECT it!

5. $2\log_4(x+4) - \log_4(x+12) = 1$

$\log_4(x+4)^2 - \log_4(x+12) = 1$

$\log_4 \frac{(x+4)^2}{x+12} = 1$

$\Rightarrow (x+12)(4^1) = \left(\frac{(x+4)^2}{\cancel{x+12}}\right) \cdot \cancel{(x+12)}$

$4x + 48 = (x+4)(x+4)$

$4x + 48 = x^2 + 4x + 4x + 16$

$0 = x^2 + 8x + 16 - 4x - 48$

$0 = x^2 + 4x - 32$

$0 = (x-4)(x+8)$

$\boxed{x = 4}$ $\cancel{x = -8}$

reject, it makes an argument negative

6. $\log_{12}(3-x) + \log_{12}(2-x) = 1$

$\log_{12}[(3-x)(2-x)] = 1$

$\log_{12}(6 - 3x - 2x + x^2) = 1$

$\log_{12}(6 - 5x + x^2) = 1$

$12^1 = 6 - 5x + x^2$

$0 = x^2 - 5x + 6 - 12$

$0 = x^2 - 5x - 6$

$0 = (x+1)(x-6)$

$\boxed{x = -1}$ $\cancel{x = 6}$

reject, it makes an argument negative

$$2^{x+3} = 16^{5x}$$
$$2^{x+3} \quad (2^4)^{5x}$$
$$2^{x+3} = 2^{20x}$$
$$\Rightarrow \cancel{x}+3 = 20\cancel{x}$$

$$19x = 3$$
$$x = 3/19$$

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Solving Exponential Equations with Different Bases

In chapter 7 we solved exponential equations by making each side of the equation use the same base. When that is hard to do, we can instead solve by taking the logarithm of each side of the original equation and solving the resulting equation.

If the bases are NOT conveniently related to each other, we should:

- **Take the logarithm of each side of the original equation**
- **Solve the resulting equation**

Example:

$$2^{x+3} = 15^{4x}$$
$$\log 2^{x+3} = \log 15^{4x}$$
$$(x+3)(\log 2) = 4x \log 15$$
$$x\log 2 + 3\log 2 = 4x \log 15$$
$$-x\log 2 \qquad -x \log 2$$
$$3\log 2 = 4x\log 15 - x\log 2$$
$$\frac{3\log 2}{(4\log 15 - \log 2)} = \frac{x\cancel{(4\log 15 - \log 2)}}{\cancel{(4\log 15 - \log 2)}}$$
$$\frac{3\log 2}{(4\log 15 - \log 2)} = x$$

Answer in exact form

$$\boxed{x \doteq 0.2051}$$

Correct to 4 decimal places

Solving Exponential Equations with Different Bases

In chapter 7 we solved exponential equations by making each side of the equation use the same base. When that is hard to do, we can instead solve by taking the logarithm of each side of the original equation and solving the resulting equation.

To Try:
Solve for x. **Solve correct to 2 decimal places.**

1. $3^x = 2800$

$$\log 3^x = \log 2800$$
$$\frac{x\log 3}{\log 3} = \frac{\log 2800}{\log 3}$$
$$\boxed{x = \frac{\log 2800}{\log 3} \doteq 7.22}$$

2. $e^x = 2$

$$\log e^x = \log 2$$
$$\frac{x\log e}{\log e} = \frac{\log 2}{\log e}$$
$$\boxed{x = \frac{\log 2}{\log e} \doteq 0.69}$$

3. $3(4^{2x+3}) = 8^{4x-2}$

$$\log 3(4^{2x+3}) = \log 8^{4x-2}$$

Log both sides

$$\log 3 + \log 4^{2x+3} = \log 8^{4x-2}$$
$$\log 3 + (2x+3)\log 4 = (4x-2)\log 8$$

Use product law & power law

$$\log 3 + 2x\log 4 + 3\log 4 = 4x\log 8 - 2\log 8$$

distribute!

$$\log 3 + 3\log 4 + 2\log 8 = 4x\log 8 - 2x\log 4$$

Collect x-terms on one side of equation

$$\log 3 + 3\log 4 + 2\log 8 = x(4\log 8 - 2\log 4)$$

factor out x

$$\frac{(\log 3 + 3\log 4 + 2\log 8)}{(4\log 8 - 2\log 4)} = x$$

divide, to isolate x

$$\boxed{x \doteq 1.70}$$

- **Work on Chapter 8 Hand-in (should be able to do #1-15**
- **Optional worksheets (posted on website):**
  - **Solving Equations Practice**
  - **More Solving Practice (Log & Exponential Equations)**

- **More equation solving practice:**

- **TB p 412: 1, 2ac, 3, 4ac, 5, 6, 7acd, 8abd, 13, 16**