Class_19 Mar 28 - Rational Expressions

Thursday, March 23, 2023 7:33 PM

Tonight's Class:

- **Working through sections 6.1, 6.2**
  - **Non-Permissible Values**
  - **Multiplying and Dividing Rational Expressions**
- **Questions from Chapter 5 hand-in assignment?**
- **Chapter 5 Test (Trigonometry) next class**

6.1 Equivalent Rational Expressions and NPVs

Focus: Determine NPVs and find equivalent forms of rational expressions

rational number – if written as decimals, they terminate or go into a repeating pattern:

can be written as fractions

0.2 $0.719719719719\ldots$ $0.\overline{719}$ -13.25

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rational expressions – fractions with polynomials in the numerator and the denominator

$$f(x) = \frac{p(x)}{q(x)}$$

can never allow this part to equal zero

example: $\frac{5x+4}{x^2-9}$

these values $\{$ $x=3$ is not allowed $3^2-9 = 9-9 = 0$

these values are "non-permissible values" NPVs

$x=3$ is not allowed $3^2-9=9-9=0$

$x=-3$ " " " $(-3)^2-9=9-9=0$

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Example 1 Determining Non-Permissible Values (NPVs)

Determine the non-permissible values for each rational expression.

a) $\frac{x^2+2}{x^2-x-6}$ **b)** $\frac{x}{x^2+1}$

or restrictions

to figure out NPVs

1) set denominator equal to zero
2) solve, to see what NPVs are

a) $\frac{x^2+2}{x^2-x-6}$

$x^2-x-6=0$ ← factor, to find the x-values

[numbers multiply to -6, and add to -1: -3, +2]

$x^2-3x+2x-6=0$

$x(x-3)+2(x-3)=0$

$(x-3)(x+2)=0$

$x-3=0$ (+3 +3) $x=3$

$x+2=0$ (-2 -2) $x=-2$

NPVs: $x\neq 3$, $x\neq -2$

b) $\frac{x}{x^2+1}$

1) Set denom = 0
2) solve for x to get NPVs

1) $x^2+1=0$ (-1 -1)

$\sqrt{x^2}=\sqrt{-1}$

$x=$ no solution

2) there are NO NPVs for this expression.

p. 521 #1

Try p521 CYU #1

2) there are NO NPVs for this expression.

Check Your Understanding

1. Determine the non-permissible values for each rational expression.

a) $\frac{5x}{x^2 - 9}$ b) $\frac{3x + 2}{x^2 - 8x + 16}$

a) $\frac{5x}{x^2-9}$

1) set denom = 0 $x^2 - 9 = 0$

$+9 \quad +9$

$\sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 3$

$x^2 - 9 = 0$

$(x-3)(x+3)=0$

$x-3=0 \quad x+3=0$

$x=3 \quad x=-3$

NPVs $x \neq -3$, $x \neq 3$

b) 1) denom = 0

$x^2 - 8x + 16 = 0$

$x^2 - 4x - 4x + 16 = 0$

mult 16, add -8

$x(x-4) - 4(x-4) = 0$

$-4, -4$

$(x-4)(x-4) = 0$

$x-4=0 \quad x-4=0$

$x=4 \quad x=4$

2) NPVs $x \neq 4$

$x^2 + 0x - 9 = 0$

mult -9, add to 0: 3, -3

$x^2 + 3x - 3x - 9 = 0$

$x(x+3) - 3(x+3) = 0$

$(x+3)(x-3) = 0$

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Example 2 **Writing Equivalent Forms of a Rational Expression**

doesn't look the same, but it's "worth the same"

Use multiplication and division to write two equivalent forms of the rational expression $\frac{3(x + 2)}{(x + 2)(x - 4)}$.

$\frac{2}{6}$, $\frac{1}{3}$ } equivalent

1) simplify, reducing the expression:

$\frac{3(x+2)}{(x+2)(x-4)}$

Before we simplify, we should list any NPVs.

$(x+2)(x-4) = 0$

NPVs $x \neq -2$

NPVs $x \neq -2$, $x \neq 4$

$(x+2)(x-4) = 0$

$x+2=0$ → $x=-2$; $x-4=0$ → $x=4$

$$\frac{3(x+2)}{(x+2)(x-4)} = \frac{3}{x-4}, \text{ where } x \neq -2,\ x \neq 4$$

NPVs always come from the original form of the expression.

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2) changing to a different form to get a common denominator with another expression:

$$\frac{3(x+2)}{(x+2)(x-4)}$$

Say we wanted to add it to this expression:

$$\frac{5x}{x+1}$$

$$\frac{3(x+2)}{(x+2)(x-4)} \cdot \frac{(x+1)}{(x+1)} + \frac{5x}{(x+1)} \cdot \frac{(x-4)}{(x-4)} \cdot \frac{(x+2)}{(x+2)}$$

★ Careful – don't cancel individual terms!!

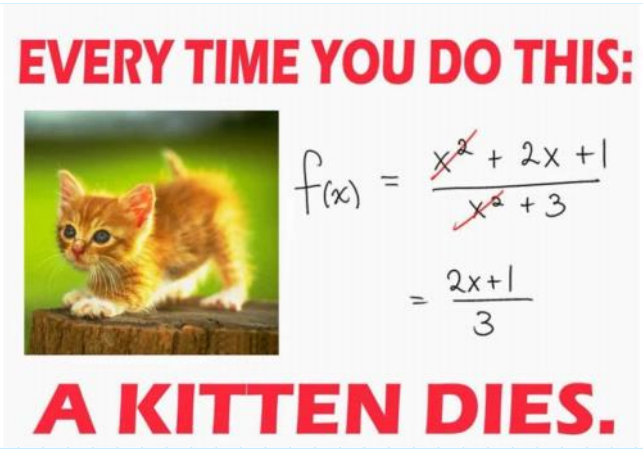
This is ok:

$$\frac{(x+4)}{(x+4)(x+5)} = \frac{1}{(x+5)}$$

This is NOT okay:

$$\frac{x+4}{x+5} = \frac{4}{5}$$

no!! no!! no!! no!!



don't do it

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Example 3 **Simplifying Rational Expressions**

Write each rational expression in simplest form.

a) $\frac{15x^2yz^3}{20xyz}$ **b)** $\frac{3x^2 - 8x}{2x}$

c) $\frac{x^2 + 6x + 8}{x^2 - 4}$ **d)** $\frac{16 - x^2}{2x^2 - 11x + 12}$

1) factor the numerator + denominator (if possible)

2) identify NPVs

3) reduce identical factors.

a) $\frac{15x^2yz^3}{20xyz}$

1) no factoring to do

2) $20xyz = 0$

$x \neq 0$
$y \neq 0$
$z \neq 0$ NPVs

$\frac{15x^2yz^3}{20xyz} = \frac{15 \cdot x \cdot y \cdot z \cdot z \cdot z}{20 \cdot x \cdot y \cdot z} = \frac{15 \div 5 \cdot x \cdot z^2}{20 \div 5} = \boxed{\frac{3xz^2}{4}}$

or, use exponent laws

b) $\frac{3x^2 - 8x}{2x}$

1) factor $\frac{3x^2 - 8x}{2x} = \frac{x(3x - 8)}{2x}$

2) NPVs: $\frac{2x}{2} = \frac{0}{2}$

$x \neq 0$ ← with this NPV

3) $\frac{x(3x-8)}{2x} = \frac{3x-8}{2}$

c) $\frac{x^2+6x+8}{x^2-4}$

1) factor $x^2+4x+2x+8$ mult to 8, add 6 → 4, 2

$x(x+4)+2(x+4)$

$(x+4)(x+2)$

$\frac{(x+4)(x+2)}{(x+2)(x-2)}$ x^2-4

2) NPVs: $x \neq -2$, $x \neq 2$

$(x+2)(x-2)=0$

$x+2=0$, $x=-2$; $x-2=0$, $x=2$

3) $\frac{(x+4)(x+2)}{(x+2)(x-2)} = \frac{x+4}{x-2}$

d) $\frac{16-x^2}{2x^2-11x+12}$

1) factor top, factor bottom

$\frac{16-x^2}{2x^2-11x+12} = \frac{(4+x)(4-x)}{(x-4)(2x-3)}$

24: 1, 24; 2, 12; 3, 8; 4, 6

AC = 24; add to -11: -8, -3

$2x^2-8x-3x+12$

$2x(x-4)-3(x-4)$

$(x-4)(2x-3)$

2) NPVs: $x \neq 4$, $x \neq \frac{3}{2}$

$x-4=0$, $x=4$

$2x-3=0$, $+3$ $+3$, $\frac{2x}{2}=\frac{3}{2}$, $x=\frac{3}{2}$

3) $\frac{(4+x)(4-x)}{(x-4)(2x-3)} = \frac{(4+x)(4-x)}{-1(-x+4)(2x-3)}$

$= \frac{(4+x)(4-x)}{-1(4-x)(2x-3)}$

$= \frac{4+x}{-1(2x-3)}$

$\frac{5}{-8}$ or $\frac{-5}{8}$ or $-\frac{5}{8}$

(4+x)

$\frac{5}{-8}$ or $\frac{-5}{8}$ or $-\frac{5}{8}$

$-1(2x-3)$

OR $\quad -\frac{(4+x)}{2x-3}$ OR $\quad -\frac{4+x}{2x-3}$

p528, #11

**Super important to always factor FULLY before you try to get the NPVs or simplify.

- Check for common factors FIRST
- Then, see if you can factor even more completely, afterward

6.2 Multiplying and Dividing Rational Expressions

Focus: Multiply and divide rational expressions

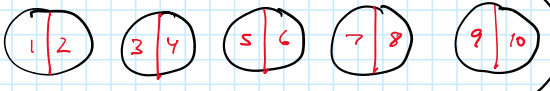
remember, with fractions:

$\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$

$\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3}$

$= \frac{5}{6}$

$5 \div \frac{1}{2} = 5 \times \frac{2}{1} = 10$



To **multiply two rational expressions**, multiply numerators and multiply denominators:

$$\frac{A}{B}\cdot\frac{C}{D}=\frac{A\cdot C}{B\cdot D}$$

It is often useful to factor the components of the rational expression so that we can cancel all common factors in the numerator and denominator.

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Example 1 **Multiplying Rational Expressions**

Simplify each expression.

a) $\frac{c^2}{10}\cdot\frac{5d}{2c}$

b) $\frac{4x(x+3)}{3(x-1)}\cdot\frac{5(x-1)}{2x}$

1) factor, if possible
2) NPVs
3) simplify

a) $\frac{c^2}{10}\cdot\frac{5d}{2c}=\frac{5c^2d}{20c}$

1) factor – none
2) NPV $c\neq 0$
3) simplify

$=\frac{\overset{1}{\cancel{5}}c^2d}{\underset{4}{\cancel{20}}c}$

$=\frac{c^2d}{4c}$

$=\boxed{\frac{cd}{4}}$

b) $\frac{4x(x+3)}{3(x-1)}\cdot\frac{5(x-1)}{2x}$

1) already factored
2) NPV $x\neq 0$, $x\neq 1$

$\frac{3(x-1)}{3}=\frac{0}{3}$

$x-1=0$

$+1 \quad +1$

$x=1$

$=\frac{4x(x+3)(5)}{6x}$

$=\frac{\overset{\div 2}{20}x(x+3)}{6\overset{\div 2}{x}}$

$$= \frac{(10)(x+3)}{3}$$

$$= \frac{10(x+3)}{3}$$

Dividing Fractions

$$\frac{3}{4} \div \frac{2}{7}$$

Keep $\frac{3}{4}$ Change $\times$ Flip $\frac{7}{2}$ $= \frac{21}{8}$

To **divide two rational expressions**, multiply by the reciprocal of the divisor (i.e., change division into multiplication by "flipping" the second fraction):

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$$

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Example 2 Dividing Rational Expressions

Simplify each expression.

a) $\frac{5n^4}{-2} \div \frac{(5n)^2}{6}$

b) $\frac{2(x+1)}{3x} \div \frac{4(x+1)}{x(x-2)}$

1) change to multiplication question
2) factor everything
3) NPVs
4) simplify

a) $\frac{5n^4}{-2} \div \frac{(5n)^2}{6}$

$= \frac{5n^4}{-2} \cdot \frac{6}{(5n)^2}$

$= \frac{5n^4}{-2} \cdot \frac{6}{25n^2}$

1) we flipped the 2nd expression
2) no factoring
3) NPVs $n \neq 0$
4) simplify
- reduce first, & then multiply if

$$= \frac{1n^4}{-1} \cdot \frac{3}{5n^2}$$

$$= \frac{3n^4}{-5n^2}$$

$$= \boxed{\frac{3n^2}{-5}}$$

you would like to keep the numbers a bit smaller.

$$\frac{5n^4}{-2} \cdot \frac{6}{25n^2} = \frac{30n^4}{-50n^2} \quad \begin{matrix} \div 10 \\ \div 10 \end{matrix}$$

$$= \frac{3n^4}{-5n^2}$$

$$= \frac{3n^2}{-5}$$

OR, you can multiply first & then reduce.

For next class

- **Work on the worktext questions for 6.1-6.2**
- **Prepare for the Chapter 5 Test**
- **Complete the Chapter 5 hand-in**