Class_19 Nov 15 - Log Graphs and Laws

Sunday, November 6, 2022 8:23 PM

Tonight's Class:

- **Chapter 7 Test**
- **8.2 Transforming Log Graphs**
- **8.3 Logarithm Laws**

Please:

1. **Make sure your name is on your Chapter 7 Hand-in and give it to me.**
2. **Put away materials, except for your calculator & something to write with.**
3. **On your test, write clearly and show all necessary steps.**
   When you are finished, please look over your test before handing it in.
4. **While other people are still finishing, respect them by being quiet.**

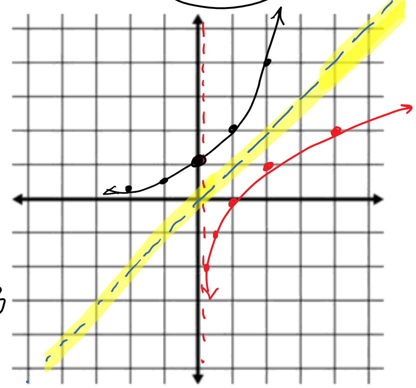
Review - work on Log Practice worksheet, #1-4 only.

Graphing an Exponential Function and its Inverse

a) Fill in the table below, and sketch the graph of the exponential function, $y = 2^x$.

$y = 2^x$

x	y	
-2	2^{-2}	$= \frac{1}{2^2} = \frac{1}{4}$
-1	2^{-1}	$= \frac{1}{2}$
0	2^0	$= 1$
1	2^1	$= 2$
2	2^2	$= 4$



b) Identify the following:

- domain $\{x \mid x \in \mathbb{R}\}$
- range $\{y \mid y > 0, y \in \mathbb{R}\}$
- asymptote equation $y = 0$
- x-intercept, if it exists none
- y-intercept, if it exists $(0, 1)$

c) Give the equation of the ***inverse*** of: $y = 2^x$. Inverse's equation is: $x = 2^y$

d) For the equation of the inverse that you found in part c), complete the table at right and sketch the graph on the grid above.

x	y
$1/4$	-2
$1/2$	-1
1	0
2	1
4	2

e) For the inverse graph, what are its:

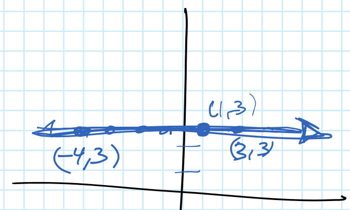
- domain $\{x \mid x > 0, x \in \mathbb{R}\}$
- range $\{y \mid y \in \mathbb{R}\}$
- asymptote equation → $x = 0$
- x-intercept, if it exists $(1, 0)$
- y-intercept, if it exists none

f) Rewrite the inverse equation from part c) in logarithmic form: $\log_2 x = y$

so, $\boxed{y = \log_2 x}$ and $\boxed{y = 2^x}$ are inverses

Conclusions: $y = a^x$ and $y = \log_a x$ are INVERSES

$\log_a(a^x) = x$ $\qquad a^{\log_a x} = x$



horizontal line

$y = 3$

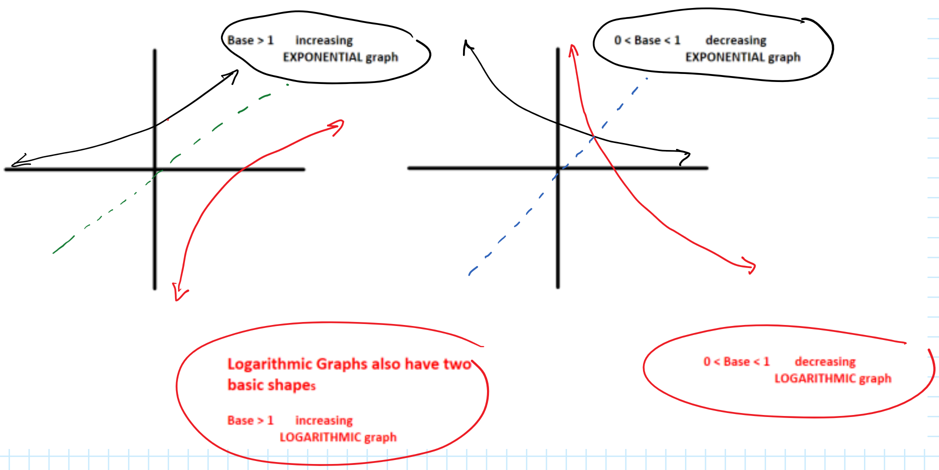
$\log_3 9 = 2$ (because $3^2 = 9$)

→ equals the exponent we NEED, in order to change the base, 3, into the argument, which is 9

$\log_a x = ?$ the exponent NEED, to change the base, a, into the argument, which is x.

Log Practice worksheet, #5 only

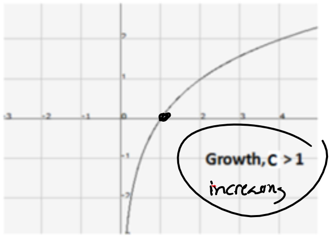
<u>Remember</u>
Exponential graphs have two basic shapes.

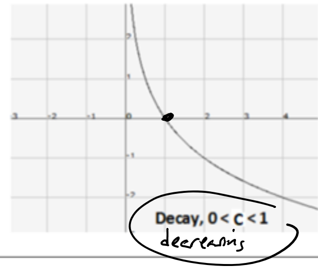


8.2 Transformations of Logarithmic Functions

The graphs of logarithmic functions can be grouped into two categories:

- if the logarithm's base is larger than one, the graph is ***increasing***
- if the logarithm's base is between zero and one, the graph is ***decreasing***





What characteristics are the ***same*** for all untransformed logarithmic graphs?

domain: $\{x \mid x > 0, x \in \mathbb{R}\}$ x-intercept $(1, 0)$

range: $\{y \mid y \in \mathbb{R}\}$ vertical asymptote: $x = 0$

Predict what will happen to the graph of $y = \log_3 x$ when each of the following changes is made to the equation:

$y = \log_3(x) - 5$ down 5 $y = \log_3(x-5)$ right 5

$y = -4\log_3 x$ VE by 4, reflect across x-axis

$y = \log_3\left(-\frac{2}{5}(x+3)\right)$ HE $5/2$, reflect across y-axis, left 3

Horizontal stretch by a factor of $\frac{1}{b}$

Vertical stretch by a factor of a

Horizontal translation

Vertical translation

$$y = a\log_c\left[b(x-h)\right] + k$$

If b < 0 then there is a reflection over the y-axis (horizontal reflection)
If a < 0 then there is a reflection over the x-axis (vertical reflection)

Example, TB p 387:

Your Turn

a) Use transformations to sketch the graph of the function $y = 2\log_3(-x+1)$.

b) Identify the following characteristics.

i) the equation of the asymptote **ii)** the domain and range

iii) the y-intercept, if it exists **iv)** the x-intercept, if it exists

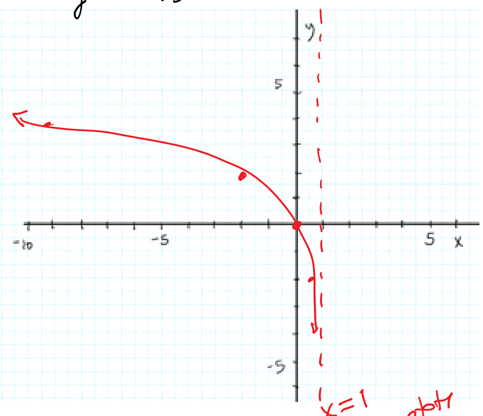
FACTOR

$y = 2\log_3(-x+1)$ **Base function** $y = \log_3 x$ x | y instead 3^x

FACTOR

$y = 2\log_3(-x+1)$

$y = 2\log_3(-1(x-1))$



$x=1$
V. asymptote

Base function $y = \log_3 x$

$x = 3^y$

x	y
$\frac{1}{3}$	$-1 = \log_3(\frac{1}{3})$
1	$0 = \log_3 1$
3	$1 = \log_3 3$
9	$2 = \log_3 9$
27	$3 = \log_3 27$

instead

$y = 3^x$

x	y
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

Transformations and mapping notation

VE by 2
reflect across y-axis
right 1

$(x, y) \rightarrow (-x+1, 2y)$

$-x+1$	$2y$
$-\frac{1}{3}+1 = \frac{2}{3}$	-2
$-1+1=0$	0
$-3+1=-2$	2
$-9+1=-8$	4
$-27+1=-26$	6

Domain $\{x \mid x<1,\ x \in \mathbb{R}\}$

Range $\{y \mid y \in \mathbb{R}\}$

Vertical asymptote equation

$x=1$

Log Practice worksheet, #6 only

Remember, all <u>untransformed</u> log graphs have
- **vertical asymptote at x = 0**
- **domain x > 0**

When a log equation is transformed, here's the an easy way to find its domain and vertical asymptote equation:

To find the domain of a logarithmic graph, either:

- **Use the fact that any untransformed log graph has domain x > 0 and apply the equation's transformations to find the new domain.**
- **Set ARGUMENT > 0, and do the algebra needed to isolate the x.**

The vertical asymptote equation uses the same number found above, and is in the form x = number.

Suppose we had this:

$(-2x+18) + 3$

Suppose we had this:

$$y = \log_4(-2x+18) + 3$$

$-2x+18 > 0$

$\frac{-2x}{-2} > \frac{-18}{-2}$

$x < 9$

$-2x+18 > 0$
$+2x \quad +2x$

$\frac{18}{2} > \frac{2x}{2}$

$9 > x$

$x = 9$

This is the vertical asymptote equation

This is the domain

Example:

$$y = \log_2(x+4) - 7$$

argument

- Domain
- Range $y \in \mathbb{R}$
- Asymptote equation $x = -4$
- *x*-intercept
- *y*-intercept

argument > 0

$x+4 > 0$

$x > -4$

x-intercept, let $y=0$

$0 = \log_2(x+4) - 7$ } isolate log term

$7 = \log_2(x+4)$

$2^7 = x+4$ } change form

$128 = x+4$

$x = 124$

$(124, 0)$

y-intercept, let $x=0$

$y = \log_2(x+4) - 7$

$y = \log_2(0+4) - 7$

$y = \log_2(4) - 7$

$y = 2 - 7$

$(0, -5)$

$y = -7$

try these

For each of the following find:

- Domain
- Range
- Asymptote equation
- *x*-intercept
- *y*-intercept

1. $y = \log_2(x-7)-5$

2. $y = \log_3(5x+3)-4$

For each of the following find:

- Domain
- Range
- Asymptote equation
- *x*-intercept
- *y*-intercept

1. $y = \log_2(x-7)-5$

2. $y = \log_3(5x+3)-4$

domain: $x-7>0$

$\{x \mid x>7,\ x \in \mathbb{R}\}$

range: $\{y \mid y \in \mathbb{R}\}$

asymptote equation: $x=7$

x-intercept: $0 = \log_2(x-7)-5$

$+5 \qquad +5$

$5 = \log_2(x-7)$

$\log_2(x-7) = 5$

$2^5 = x-7$

$32 = x-7$, $x=39$

$\boxed{(39,0)}$

y-intercept:

$y = \log_2(0-7)-5$

$y = \log_2(-7)-5$

no y-intercept

domain: $5x+3>0$

$5x>-3$

$\boxed{x>-\frac{3}{5}}$

range: $\boxed{y \in \mathbb{R}}$

asymp: $\boxed{x=-\frac{3}{5}}$

x-int:

$0 = \log_3(5x+3)$

$4 = \log_3(5x+3$

$3^4 = 5x+3$

$81 = 5x+3$

$\frac{78}{5} = \frac{5x}{5}$

$x = \frac{78}{5} = 1$

$\boxed{(1,0)}$

y-int:

$y = \log_3(5x+3)-4$

$y = \log_3(3)-4$

$y = 1-4$

$y=-3$

$\boxed{(0,-3)}$

Investigate - on whiteboards

Logarithms – Investigation

Part I:

Evaluate the expressions on the left, using your understanding of logs.
Re-write each of your answers as a single logarithm, as shown in the example.

Example: $\log_4(16)+\log_4(4)=2+1=3$ This answer, 3, is equal to $\log_4(64)$

We've shown that: $\log_4(16)+\log_4(4)=\log_4(64)$

1) $\log_2(8)+\log_2(4)=$ _____ $\log_2(\quad)$

2) $\log_3(9)+\log_3(81)=$ _____ $\log_3(\quad)$

these can be written as...

3) $\log_3\left(\frac{1}{9}\right)+\log_3(81)=$ _____ $\log_3(\quad)$

4) $\log_5(5)+\log_5(1)=$ _____ $\log_5(\quad)$

5) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X+\log_c Y=\log_c(\qquad)}$$

Below are some questions that we can't answer directly with the definition of logarithms. Use the pattern discovered above to write each one as a single logarithm, then evaluate it.

6) $\log_6 12+\log_6 3$

7) $\log 250+\log 40$

8) $\log_8\left(\frac{3}{64}\right)+\log_8\left(\frac{1}{3}\right)$

Part II:

Evaluate the expressions on the left, using your understanding of logs.
Re-write each of your answers as a single logarithm, as shown in the example.

Example: $\log_4(64) - \log_4(16) = 3 - 2 = 1$ This answer, 1, is equal to $\log_4(4)$

We've shown that: $\log_4(64) - \log_4(16) = \log_4(4)$

9) $\log_5 625 - \log_5 5 =$ ______ $\log_5(\quad)$

10) $\log_6 36 - \log_6 6 =$ ______ $\log_6(\quad)$

which is the same as...

11) $\log_3 9 - \log_3 1 =$ ______ $\log_3(\quad)$

12) $\log_2 16 - \log_2 32 =$ ______ $\log_2(\quad)$

13) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X - \log_c Y = \log_c(\qquad)}$$

Below are more questions that we can't answer directly with the definition of logarithms. Use the new pattern discovered above to write each one as a single logarithm, then evaluate it.

14) $\log_6 72 - \log_6 2$

15) $\log 12 - \log 0.12$

16) $\log_{12} 2 - \log_{12} 288$

Hand-in Worksheet: Chapter 8 Hand-in
Should be okay to do #1-7 right now.

Practice
(8.2) TB p 389: 1, 2, 4c, 5c, 6, 7, 8ab, 9b, 13
(8.3) TB p 400: 1-5, 8-10
(8.4) TB p 412: 1, 3, 4ac, 5, 6, 8abd

Lots of detailed, careful solutions of logarithmic equations found here:
https://www.chilimath.com/lessons/advanced-algebra/solving-logarithmic-equations/

Unit 3 Test next Tuesday, Nov 22

Study Suggestions:

- **Work on Chapter 8 Hand-in (#1-7, for now)**

- **Complete optional Worksheets (posted on website):**
  - **Unit 3 Solving Equations Practice**
  - **More Solving Practice (Log & Exponential Equations)**
  - **Chapter 8 Review**

- **Equation solving:**
  - **TB p 412: 1, 2ac, 3, 4ac, 5, 6, 7acd, 8abd, 13, 16**