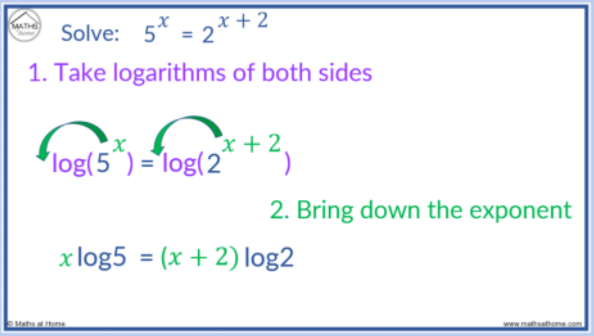
Class_20 June 7 Solving Log & Exponential Equations & Applications

Plan For Today:

1. Question about anything from last class? 8.3-8.4
2. Finish Chapter 8: Logarithmic Functions
    - ✓ 8.1: Understanding Logarithms
    - ✓ 8.2: Transformations of Logarithmic Functions
    - ✓ 8.3: Laws of Logarithms
    - ❖ **8.4: Logarithmic & Exponential Functions**
5. Work on practice questions from Textbook

Page 412:
#1, 2ac, 3, 4ac, 5, 6, 7acd, 8abd, 13, 15, 16, 17



Solve: $5^x = 2^{x+2}$

1. Take logarithms of both sides

$\log(5^x) = \log(2^{x+2})$

2. Bring down the exponent

$x\log 5 = (x+2)\log 2$

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Plan Going Forward:

1. Finish working through extra practice & textbook questions from 8.4 and finish working on the Ch. 8 Assignment.

- ❖ **CHAPTER 8 ASSIGNMENT DUE ON MONDAY, JUNE 12TH**

2. You will start ch9 (9.1-9.2) tomorrow. Ch9 will be covered next Monday after the test and we will plan to finish it on Tuesday. Last topic 10 will start next Wednesday.

- ❖ **TEST 6 ON 8.2-9.2 ON MONDAY, JUNE 12TH**
- ❖ **TEST 7 ON 9.3-10.4 ON MONDAY, JUNE 12TH**
- ❖ **CHAPTER 9 ASSIGNMENT DUE ON THURSDAY, JUNE 15TH**
- ❖ **TOPIC 10 (G) ASSIGNMENT DUE ON TUESDAY, JUNE 20TH**

Please let us know if you have any questions or concerns about your progress in this course. The notes from today will be posted at egolfmath.weebly.com after class.

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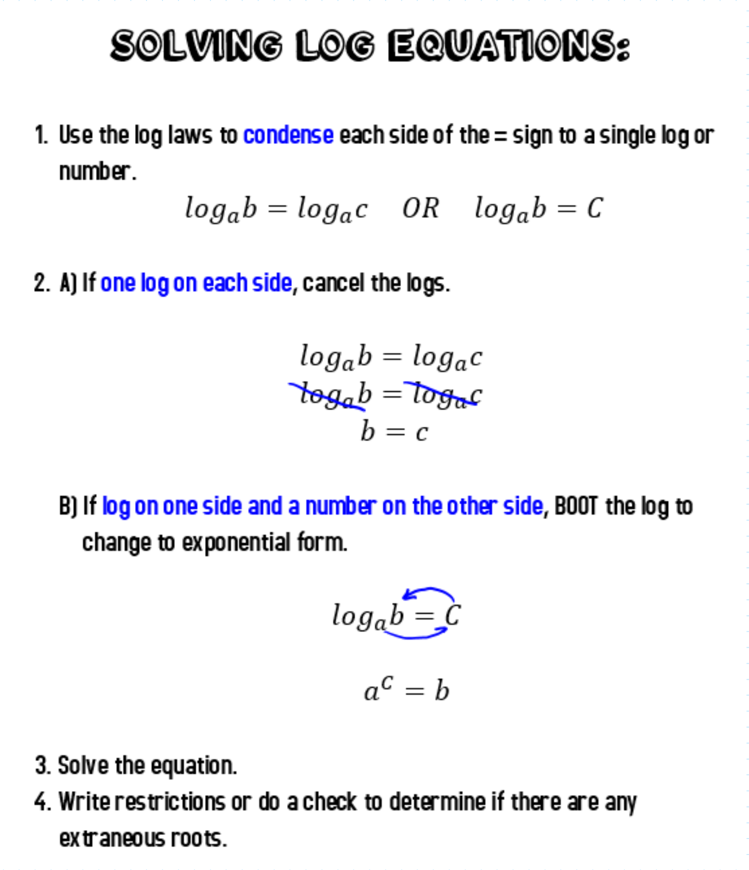
8.4: Review Solving Logarithmic & Exponential Functions

Using log laws to solve the log equation:



SOLVING LOG EQUATIONS:

1. Use the log laws to **condense** each side of the = sign to a single log or number.

$$log_a b = log_a c \quad OR \quad log_a b = C$$

2. A) If **one log on each side**, cancel the logs.

$$log_a b = log_a c$$
$$\cancel{log_a} b = \cancel{log_a} c$$
$$b = c$$

B) If **log on one side and a number on the other side**, BOOT the log to change to exponential form.

$$log_a b = C$$

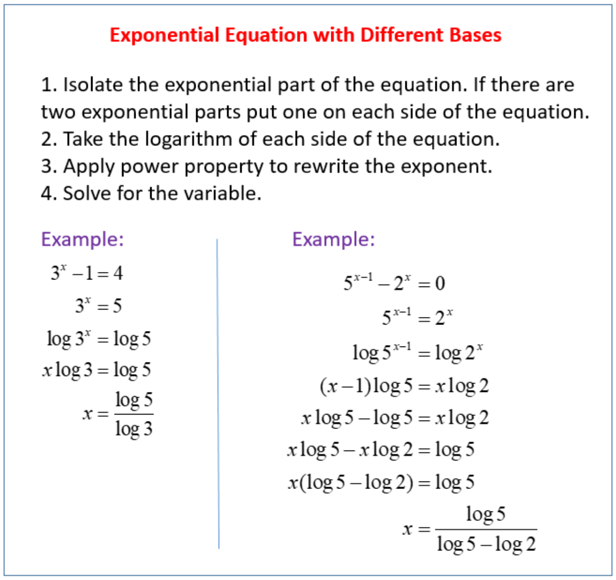
$$a^C = b$$

3. Solve the equation.
4. Write restrictions or do a check to determine if there are any extraneous roots.

Solving a logarithmic equation by changing to exponential form = BOOT THE LOG

Solve the log equation by combining to a common of base:

Recall solving an exponential by changing to a common base. You can then make the exponents equal and solve.



Exponential Equation with Different Bases

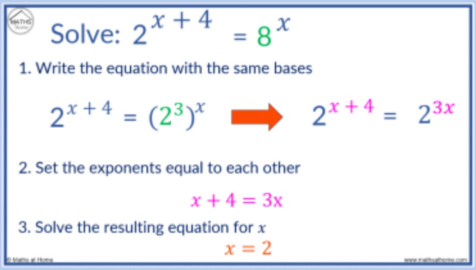
1. Isolate the exponential part of the equation. If there are two exponential parts put one on each side of the equation.
2. Take the logarithm of each side of the equation.
3. Apply power property to rewrite the exponent.
4. Solve for the variable.

Example:

$$3^x - 1 = 4$$
$$3^x = 5$$
$$\log 3^x = \log 5$$
$$x \log 3 = \log 5$$
$$x = \frac{\log 5}{\log 3}$$

Example:

$$5^{x-1} - 2^x = 0$$
$$5^{x-1} = 2^x$$
$$\log 5^{x-1} = \log 2^x$$
$$(x-1)\log 5 = x \log 2$$
$$x \log 5 - \log 5 = x \log 2$$
$$x \log 5 - x \log 2 = \log 5$$
$$x(\log 5 - \log 2) = \log 5$$
$$x = \frac{\log 5}{\log 5 - \log 2}$$



Solve: $2^{x+4} = 8^x$

1. Write the equation with the same bases

$$2^{x+4} = (2^3)^x \rightarrow 2^{x+4} = 2^{3x}$$

2. Set the exponents equal to each other

$$x + 4 = 3x$$

3. Solve the resulting equation for x

$$x = 2$$

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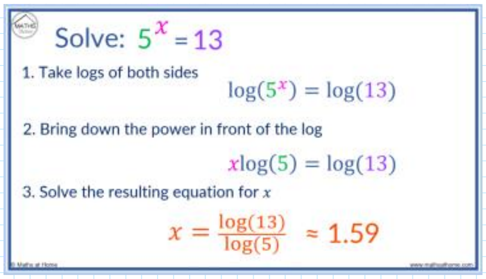

Solve: $5^x = 13$

1. Take logs of both sides

$$\log(5^x) = \log(13)$$

2. Bring down the power in front of the log

$$x\log(5) = \log(13)$$

3. Solve the resulting equation for x

$$x = \frac{\log(13)}{\log(5)} \approx 1.59$$

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What if you can't get a common base? Log both side and use the power law to solve.

SOLVING EXPONENTIAL EQUATIONS WITH LOGS:

1. **Simplify equation by trying to get a single base on both sides of the equal sign.**

$$M^{a+b} = N^{c+d}$$

Recall: if there is a single common base on each side of the = sign, cancel the bases and make the exponents equal to solve.

$$M^{a+b} = M^{c+d}$$

$$\cancel{M}^{a+b} = \cancel{M}^{c+d}$$

$$a + b = c + d$$

2. **If you cannot get a common base, take the log of both sides.**

$$\log M^{a+b} = \log N^{c+d}$$

3. **Use the power law to bring the exponent to the front of the log.**

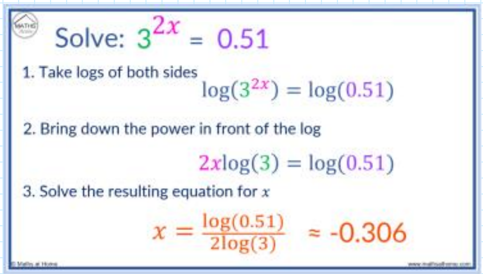
$$\log M^{a+b} = \log N^{c+d}$$

$$(a + b)\log M = (c + d)\log N$$

4. **Expand the brackets by distribution, collect the common variables to one side, factor and solve for x.**

$$(a + b)\log M = (c + d)\log N$$

$$a\log M + b\log M = c\log N + d\log N$$



Solve: $3^{2x} = 0.51$

1. Take logs of both sides

$$\log(3^{2x}) = \log(0.51)$$

2. Bring down the power in front of the log

$$2x\log(3) = \log(0.51)$$

3. Solve the resulting equation for x

$$x = \frac{\log(0.51)}{2\log(3)} \approx -0.306$$

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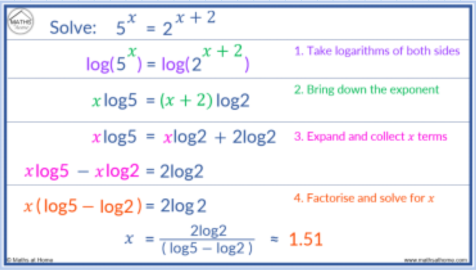

Solve: $5^x = 2^{x+2}$

$\log(5^x) = \log(2^{x+2})$ 1. Take logarithms of both sides

$x\log5 = (x + 2)\log2$ 2. Bring down the exponent

$x\log5 = x\log2 + 2\log2$ 3. Expand and collect x terms

$x\log5 - x\log2 = 2\log2$

$x(\log5 - \log2) = 2\log2$ 4. Factorise and solve for x

$x = \frac{2\log2}{(\log5 - \log2)} \approx 1.51$

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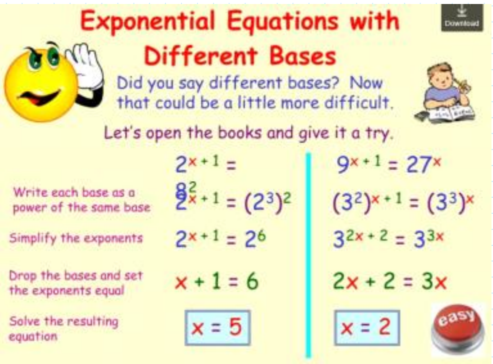
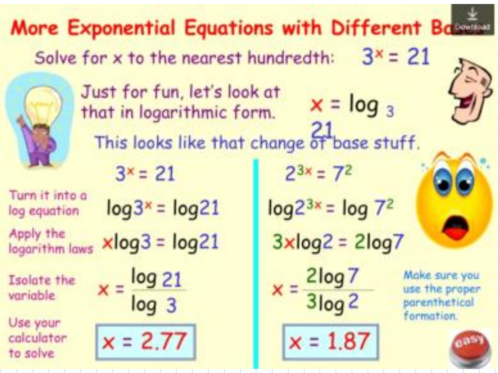


Exponential Equations with Different Bases

Did you say different bases? Now that could be a little more difficult.

Let's open the books and give it a try.

	$2^{x+1} =$ 8^2	$9^{x+1} = 27^x$
Write each base as a power of the same base	$2^{x+1} = (2^3)^2$	$(3^2)^{x+1} = (3^3)^x$
Simplify the exponents	$2^{x+1} = 2^6$	$3^{2x+2} = 3^{3x}$
Drop the bases and set the exponents equal	$x + 1 = 6$	$2x + 2 = 3x$
Solve the resulting equation	$\boxed{x = 5}$	$\boxed{x = 2}$



More Exponential Equations with Different Ba

Solve for x to the nearest hundredth: $3^x = 21$

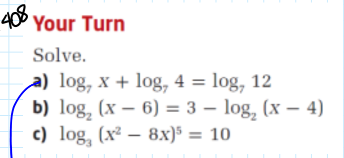
Just for fun, let's look at that in logarithmic form. $x = \log_3 21$

This looks like that change of base stuff.

	$3^x = 21$	$2^{3x} = 7^2$
Turn it into a log equation	$\log 3^x = \log 21$	$\log 2^{3x} = \log 7^2$
Apply the logarithm laws	$x\log 3 = \log 21$	$3x\log 2 = 2\log 7$
Isolate the variable	$x = \frac{\log 21}{\log 3}$	$x = \frac{2\log 7}{3\log 2}$
Use your calculator to solve	$\boxed{x = 2.77}$	$\boxed{x = 1.87}$

Make sure you use the proper parenthetical formation.

p408



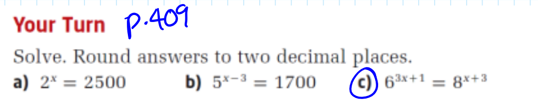
Your Turn

Solve.

a) $\log_7 x + \log_7 4 = \log_7 12$

b) $\log_2 (x - 6) = 3 - \log_2 (x - 4)$

c) $\log_3 (x^2 - 8x)^5 = 10$



Your Turn p.409

Solve. Round answers to two decimal places.

a) $2^x = 2500$ **b)** $5^{x-3} = 1700$ **c)** $6^{3x+1} = 8^{x+3}$

a) $\log_7 x + \log_7 4 = \log_7 12$ ① product law

$\cancel{\log_7}(4x) = \cancel{\log_7} 12$ ② cancel logs

$\frac{4x}{4} = \frac{12}{4}$

$\boxed{x = 3}$ ✓

③ check. Restrictions

$\log_7 x \rightarrow \therefore x > 0$

b) $\log_2 (x-6) = 3 - \log_2 (x-4)$

$+\log_2(x-4)$

① move logs to same side THEN product law

$\log_2(x-6) + \log_2(x-4) = 3$

$\log_2 (x-6)(x-4) = 3$

② Boot base + change to exponential form (remove log)

$(x-6)(x-4) = 2^3$

$x^2 - 10x + 24 = 8$

-8

③ solve
- ↳ expand
- ↳ collect like terms
- ↳ make $=0$
- ↳ factor or if not factorable, quad. formula.

$x^2 - 10x + 16 = 0$

$(x-8)(x-2) = 0$

$\boxed{x = 8}$, $x \neq 2$ extraneous.

④ Restrictions.

$x - 6 > 0$ $x - 4 > 0$

$\underline{x > 6}$ $x > 4$

c) $\log 6^{3x+1} = \log 8^{x+3}$

① no common base b/w 6 + 8 ∴ log both sides

② Power law → move exponent to front of log ☆ in a bracket

$(3x+1)\log 6 = (x+3)\log 8$

③ Expand brackets + multiply with log terms ☆ write term in front of log

$3x\log 6 + \log 6 = x\log 8 + 3\log 8$

$-x\log 8$ $-\log 6$

④ x log terms to one side + other (constants) to other side.

$3x\log 6 - x\log 8 = 3\log 8 - \log 6$

GCF $= x$

⑤ Factor x + solve (divide)

$\frac{x(3\log 6 - \log 8)}{(\div)} = \frac{3\log 8 - \log 6}{(\quad)}$

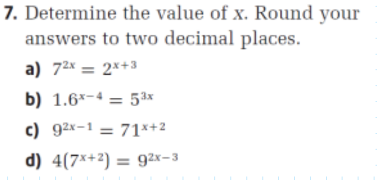
$\boxed{x = \frac{3\log 8 - \log 6}{3\log 6 - \log 8}}$ exact → Note: same answer if all signs are opposite.

$x = \frac{\log 6 - 3\log 8}{\log 8 - 3\log 6}$

$x \approx 2.43$
1.35 ←
1.60

$\boxed{x \approx 1.35}$ approximate.

p.413



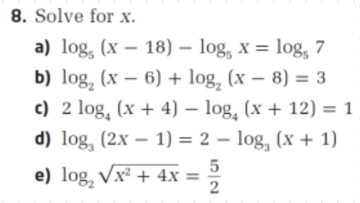
7. Determine the value of x. Round your answers to two decimal places.

a) $7^{2x} = 2^{x+3}$

b) $1.6^{x-4} = 5^{3x}$

c) $9^{2x-1} = 71^{x+2}$

d) $4(7^{x+2}) = 9^{2x-3}$



8. Solve for x.

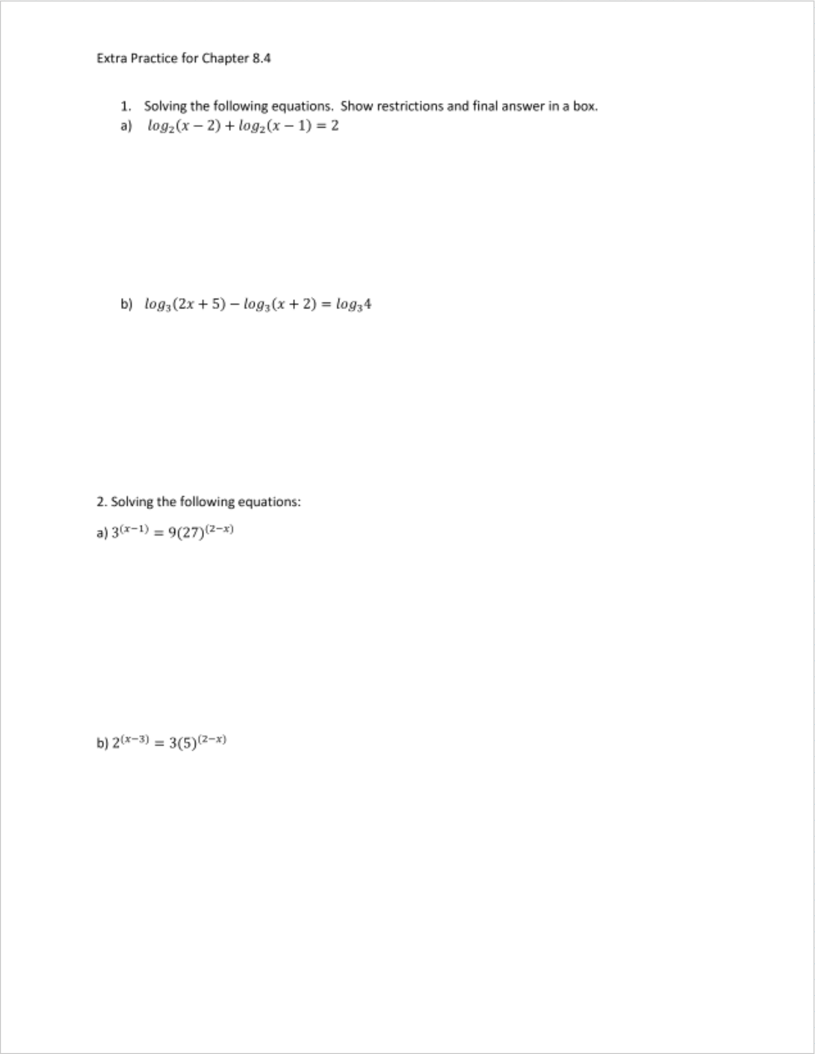
a) $\log_5 (x - 18) - \log_5 x = \log_5 7$

b) $\log_2 (x - 6) + \log_2 (x - 8) = 3$

c) $2 \log_4 (x + 4) - \log_4 (x + 12) = 1$

d) $\log_3 (2x - 1) = 2 - \log_3 (x + 1)$

e) $\log_2 \sqrt{x^2 + 4x} = \frac{5}{2}$



Extra Practice for Chapter 8.4

1. Solving the following equations. Show restrictions and final answer in a box.

a) $log_2(x-2)+log_2(x-1)=2$

b) $log_3(2x+5)-log_3(x+2)=log_3 4$

2. Solving the following equations:

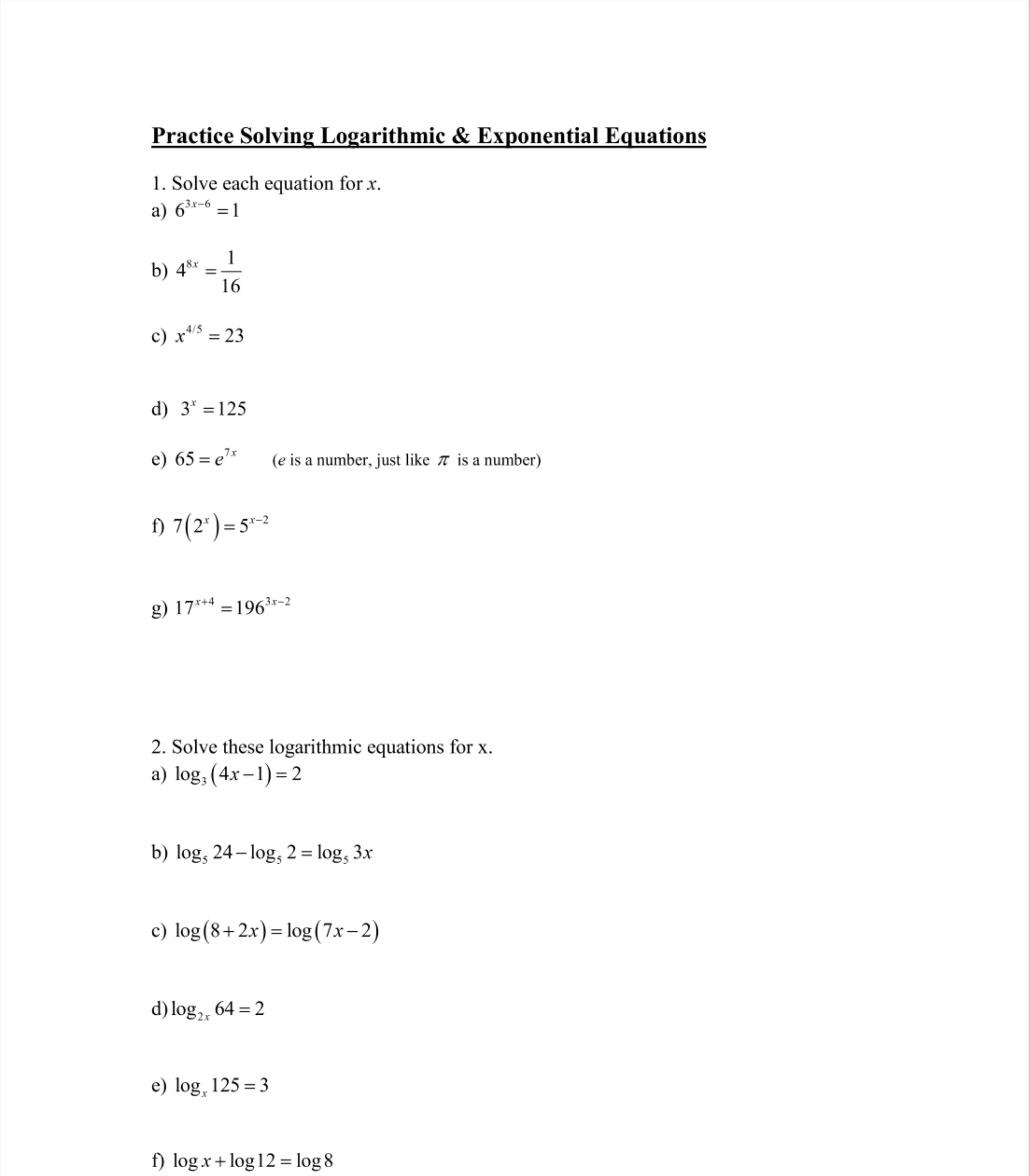
a) $3^{(x-1)}=9(27)^{(2-x)}$

b) $2^{(x-3)}=3(5)^{(2-x)}$

C_21 Key and More Solving Practice

Wednesday, November 13, 2019 4:49 PM

C_21 More Solving Practice with Solutions



Practice Solving Logarithmic & Exponential Equations

1. Solve each equation for x.

a) $6^{3x-6}=1$

b) $4^{8x}=\frac{1}{16}$

c) $x^{4/5}=23$

d) $3^{x}=125$

e) $65=e^{7x}$ (e is a number, just like π is a number)

f) $7\left(2^{x}\right)=5^{x-2}$

g) $17^{x+4}=196^{3x-2}$

2. Solve these logarithmic equations for x.

a) $\log_{3}\left(4x-1\right)=2$

b) $\log_{5}24-\log_{5}2=\log_{5}3x$

c) $\log\left(8+2x\right)=\log\left(7x-2\right)$

d) $\log_{2x}64=2$

e) $\log_{x}125=3$

f) $\log x+\log 12=\log 8$

f) $\log x + \log 12 = \log 8$

3. Solve these logarithmic equations for x.

a) $x\log 26 = \log 13$

b) $\log(5x+4) = 3$

c) $\log_4 188 = x$

d) $\log 42 = \log 14 - \log x$

e) $\ln x - \ln 4 = \ln 5$ ("ln" means $\log_e$)

f) $\ln x - \ln 4 = 5$ (This is NOT the same question as part e)

g) $\log_2\left(x^2+8\right) - \log_2 6 = \log_2 x$

h) $\log_5(3x+1) + \log_5(x-3) = 3$

i) $\log_2(x-2) + \log_2 x = \log_2 3$

j) $\log_5(x-6) = 1 - \log_5(x-2)$

k) $2\log_3 x - \log_3(x+3) - 3 = 0$

l) $\log_5(x+1) + \log_5(x-3) = 1$

Solutions

1. Solve each equation for x.

a) $6^{3x-6}=1$

$$6^{3x-6}=6^{0}$$

$$\Rightarrow 3x-6=0$$

$$3x=6$$

$$\boxed{x=2}$$

OR

$$\log 6^{3x-6}=\log 1$$

$$(3x-6)\log 6=\log 1$$

$$3x\log 6-6\log 6=\log 1$$

$$3x\log 6=\log 1+6\log 6$$

$$x(3\log 6)=\log 1+6\log 6$$

$$x=\frac{\log 1+6\log 6}{(3\log 6)}=\boxed{2}$$

b) $4^{8x}=\frac{1}{16}$

$$4^{8x}=\frac{1}{4^{2}}$$

$$4^{8x}=4^{-2}$$

$$\Rightarrow 8x=-2$$

$$x=-2/8$$

$$\boxed{x=-1/4}$$

OR

$$\log 4^{8x}=\log 1/16$$

$$8x\log 4=\log 1/16$$

$$x(8\log 4)=\log 1/16$$

$$x=\frac{\log 1/16}{(8\log 4)}$$

$$\boxed{x=-1/4}$$

c) $x^{4/5}=23$

$$\left(x^{4/5}\right)^{5/4}=(23)^{5/4}$$

$$\boxed{x\doteq 50.37}$$

d) $3^{x}=125$

$$\log 3^{x}=\log 125$$

$$x\log 3=\log 125$$

$$x=\frac{\log 125}{\log 3}$$

$$\boxed{x\doteq 4.39}$$

OR

Change form:

$$\log_3 125=x$$

$$x=\frac{\log 125}{\log 3}$$

(change of Base Law)

$$\boxed{x\doteq 4.39}$$

On a TI-83, it's found by doing $\boxed{2^{nd}}$ $\boxed{\div}$

e) $65 = e^{7x}$ (e is a number, just like π is a number) OR

$$\log 65 = \log e^{7x}$$
$$\log 65 = 7x \log e$$
$$\log 65 = x(7 \log e)$$
$$x = \frac{\log 65}{(7 \log e)} \doteq \boxed{0.60}$$

Change form

$$\log_e 65 = 7x$$
$$\frac{\log 65}{\log e} = 7x$$
$$x = \frac{\left(\frac{\log 65}{\log e}\right)}{7} \doteq \boxed{0.60}$$

By the way,

$\log_e 65$

$= LN\ 65$,

so a faster way to evaluate is

$x = \frac{LN\ 65}{7}$

$x \doteq 0.60$

f) $7(2^x) = 5^{x-2}$

$$\log\left[7(2^x)\right] = \log\left(5^{x-2}\right)$$
$$\log 7 + \log 2^x = (x-2)\log 5$$
$$\log 7 + \underline{x \log 2} = \underline{x \log 5} - 2\log 5$$
$$x\log 2 - x\log 5 = -2\log 5 - \log 7$$
$$x(\log 2 - \log 5) = -2\log 5 - \log 7$$
$$x = \frac{(-2\log 5 - \log 7)}{(\log 2 - \log 5)} \doteq \boxed{5.64}$$

g) $17^{x+4} = 196^{3x-2}$

$$\log 17^{x+4} = \log 196^{3x-2}$$
$$(x+4)\log 17 = (3x-2)\log 196$$
$$\underline{x\log 17} + 4\log 17 = \underline{3x\log 196} - 2\log 196$$
$$x\log 17 - 3x\log 196 = -2\log 196 - 4\log 17$$
$$x(\log 17 - 3\log 196) = -2\log 196 - 4\log 17$$
$$x = \frac{(-2\log 196 - 4\log 17)}{(\log 17 - 3\log 196)} \doteq \boxed{1.68}$$

$(\log 17 - 3\log 196)$

2. Solve these logarithmic equations for x.

a) $\log_3(4x-1)=2$

$$3^2 = 4x-1$$

$$9 = 4x-1$$

$$\frac{10}{4} = \frac{4x}{4} \qquad x = \frac{10}{4} = \boxed{\frac{5}{2}}$$

b) $\log_5 24 - \log_5 2 = \log_5 3x$

$$\log_5\left(\frac{24}{2}\right) = \log_5 3x$$

$$\log_5(12) = \log_5 3x$$

$$\Rightarrow \frac{12}{3} = \frac{3x}{3} \qquad \boxed{x = 4}$$

c) $\log(8+2x)=\log(7x-2)$

$$\Rightarrow 8+2x = 7x-2$$

$$\frac{-5x}{-5} = \frac{-10}{-5}$$

$$\boxed{x = 2}$$

d) $\log_{2x} 64 = 2$

$$(2x)^2 = 64$$

$$\frac{4x^2}{4} = \frac{64}{4}$$

$$x^2 = 16$$

$x = \pm 4$, but only $\boxed{x = 4}$ is valid, since base must be > 0.

e) $\log_x 125 = 3$

$\sqrt[3]{x^3} = \sqrt[3]{125}$

$\boxed{x = 5}$

f) $\log x + \log 12 = \log 8$

$\log(12x) = \log 8$

$\Rightarrow \frac{12x}{12} = \frac{8}{12}$

$x = \frac{8}{12}$, $\boxed{x = \frac{2}{3}}$

3. Solve these logarithmic equations for x.

a) $x \log 26 = \log 13$

$x = \frac{\log 13}{\log 26}$

$\boxed{x \doteq 0.79}$

b) $\log(5x+4) = 3$

$10^3 = 5x+4$

$1000 = 5x+4$

$996 = 5x$

$x = \frac{996}{5}$

$\boxed{x = 199.2}$

c) $\log_4 188 = x$

$x = \frac{\log 188}{\log 4}$ (Change of base law)

$\boxed{x \doteq 3.78}$

d) $\log 42 = \log 14 - \log x$

$$\log 42 = \log\left(\frac{14}{x}\right)$$

$$\Rightarrow \quad 42 = \frac{14}{x}$$

$$\frac{42x}{42} = \frac{14}{42}$$

$$x = \frac{14}{42}$$

$$\boxed{x = \tfrac{1}{3}}$$

e) $\ln x - \ln 4 = \ln 5$ ("ln" means $\log_e$)

$$\ln\left(\frac{x}{4}\right) = \ln 5$$

$$\Rightarrow \quad \frac{x}{4} = 5$$

$$\boxed{x = 20}$$

f) $\ln x - \ln 4 = 5$ (This is NOT the same question as part e)

$$\ln\left(\frac{x}{4}\right) = 5$$

$$e^5 = \frac{x}{4}$$

$$x = 4e^5 ,$$

$$\boxed{x \doteq 593.65}$$

g) $\log_2\left(x^2+8\right) - \log_2 6 = \log_2 x$

$$\log_2\left(\frac{x^2+8}{6}\right) = \log_2 x$$

$$\Rightarrow \quad \frac{x^2+8}{6} = x$$

$$x^2+8 = 6x$$

$$x^2-6x+8 = 0$$

$$(x-2)(x-4) = 0$$

$$\boxed{x = 2 , \quad x = 4}$$

Both are valid answers.

h) $\log_5(3x+1)+\log_5(x-3)=3$

$\log_5[(3x+1)(x-3)] = 3$

$\log_5(3x^2-9x+x-3)=3$

$5^3 = 3x^2-8x-3$

$0 = 3x^2-8x-3-125$

$3x^2-8x-128=0$

$x=\frac{8\pm\sqrt{64-(4)(3)(-128)}}{2(3)}$

$x=\frac{8\pm\sqrt{64+1536}}{6}$

$x=\frac{8\pm\sqrt{1600}}{6}$

$x=\frac{8\pm 40}{6} \rightarrow x=\frac{48}{6}=\boxed{8}$

$\rightarrow x=\frac{32}{6}=\cancel{-\frac{16}{3}}$

reject, extraneous root

i) $\log_2(x-2)+\log_2 x=\log_2 3$

$\log_2[(x-2)(x)]=\log_2 3$

$\log_2(x^2-2x)=\log_2 3$

$\Rightarrow x^2-2x=3$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$\cancel{x=-1}$ reject, extraneous

$\boxed{x=3}$ valid

j) $\log_5(x-6)=1-\log_5(x-2)$

$\log_5(x-6)+\log_5(x-2)=1$

$\log_5[(x-6)(x-2)]=1$

$\log_5(x^2-2x-6x+12)=1$

$5^1=x^2-8x+12$

$0=x^2-8x+7$

$0=(x-1)(x-7)$

$\cancel{x=1}$ reject

$\boxed{x=7}$ valid

k) $2\log_3 x-\log_3(x+3)-3=0$

$$\log_3 x^2-\log_3(x+3)=3$$

$$\log_3\left(\frac{x^2}{x+3}\right)=3$$

$$3^3=\frac{x^2}{x+3}$$

$$27=\frac{x^2}{x+3}$$

$$27(x+3)=x^2$$

$$27x+81=x^2$$

$$0=x^2-27x-81$$

$$x=\frac{27\pm\sqrt{(-27)^2-(4)(1)(-81)}}{2(1)}$$

$$x=\frac{27\pm\sqrt{1053}}{2}$$

$$\to\boxed{29.72}$$

$$\to\cancel{-2.72}$$

l) $\log_5(x+1)+\log_5(x-3)=1$

$$\log_5\left[(x+1)(x-3)\right]=1$$

$$\log_5(x^2-3x+1x-3)=1$$

$$5^1=x^2-2x-3$$

$$0=x^2-2x-8$$

$$0=(x+2)(x-4)$$

$$\cancel{x=-2}$$

$$\boxed{x=4}$$

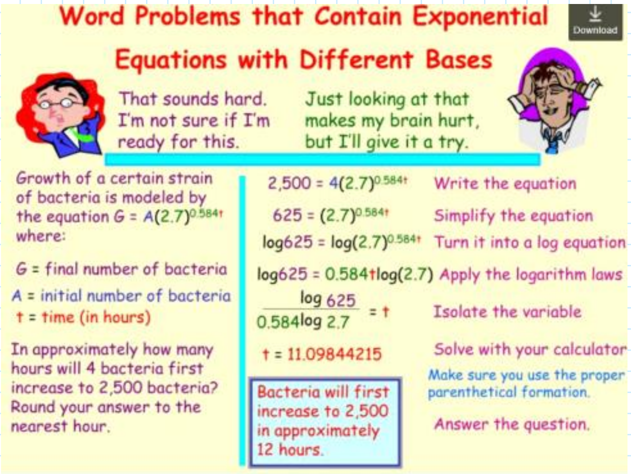
8.4 Applications

Revisiting Exponential Application Questions & Formulae

Compound Interest: $A = A_0\left(1+\frac{r}{n}\right)^{nt}$

General Growth/Decay: $A = A_0\left(b\right)^{\frac{t}{n}}$

General Earthquake/pH: $I = (10)^{high-low}$



Word Problems that Contain Exponential Equations with Different Bases

That sounds hard. I'm not sure if I'm ready for this.

Just looking at that makes my brain hurt, but I'll give it a try.

Growth of a certain strain of bacteria is modeled by the equation $G = A(2.7)^{0.584t}$ where:

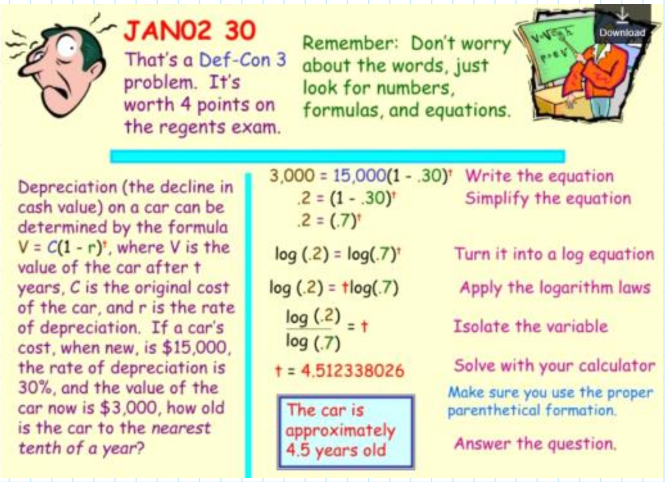
G = final number of bacteria
A = initial number of bacteria
t = time (in hours)

In approximately how many hours will 4 bacteria first increase to 2,500 bacteria? Round your answer to the nearest hour.

$2{,}500 = 4(2.7)^{0.584t}$ Write the equation
$625 = (2.7)^{0.584t}$ Simplify the equation
$\log625 = \log(2.7)^{0.584t}$ Turn it into a log equation
$\log625 = 0.584t\log(2.7)$ Apply the logarithm laws
$\frac{\log 625}{0.584\log 2.7} = t$ Isolate the variable
$t = 11.09844215$ Solve with your calculator

Make sure you use the proper parenthetical formation.

Bacteria will first increase to 2,500 in approximately 12 hours. Answer the question.



JAN02 30

That's a Def-Con 3 problem. It's worth 4 points on the regents exam.

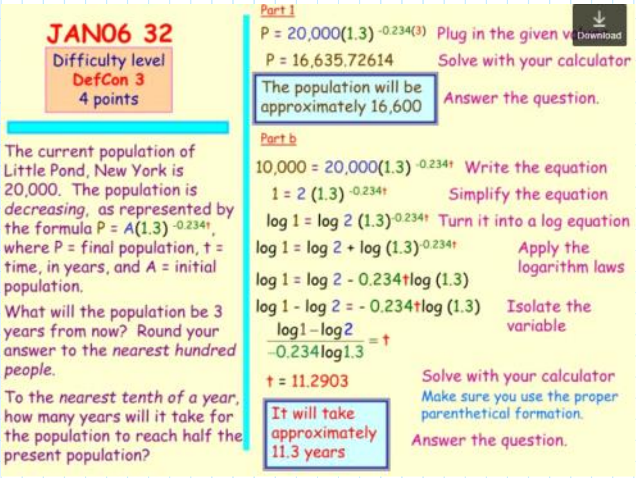
Remember: Don't worry about the words, just look for numbers, formulas, and equations.

Depreciation (the decline in cash value) on a car can be determined by the formula $V = C(1 - r)^t$, where V is the value of the car after t years, C is the original cost of the car, and r is the rate of depreciation. If a car's cost, when new, is $15,000, the rate of depreciation is 30%, and the value of the car now is $3,000, how old is the car to the *nearest tenth of a year?*

$3{,}000 = 15{,}000(1 - .30)^t$ Write the equation
$.2 = (1 - .30)^t$ Simplify the equation
$.2 = (.7)^t$
$\log(.2) = \log(.7)^t$ Turn it into a log equation
$\log(.2) = t\log(.7)$ Apply the logarithm laws
$\frac{\log(.2)}{\log(.7)} = t$ Isolate the variable
$t = 4.512338026$ Solve with your calculator

Make sure you use the proper parenthetical formation.

The car is approximately 4.5 years old Answer the question.



JAN06 32

Difficulty level
DefCon 3
4 points

The current population of Little Pond, New York is 20,000. The population is *decreasing*, as represented by the formula $P = A(1.3)^{-0.234t}$, where P = final population, t = time, in years, and A = initial population.

What will the population be 3 years from now? Round your answer to the *nearest hundred people*.

To the *nearest tenth of a year*, how many years will it take for the population to reach half the present population?

Part 1

$P = 20{,}000(1.3)^{-0.234(3)}$ Plug in the given v
$P = 16{,}635.72614$ Solve with your calculator

The population will be approximately 16,600 Answer the question.

Part b

$10{,}000 = 20{,}000(1.3)^{-0.234t}$ Write the equation
$1 = 2(1.3)^{-0.234t}$ Simplify the equation
$\log 1 = \log 2(1.3)^{-0.234t}$ Turn it into a log equation
$\log 1 = \log 2 + \log(1.3)^{-0.234t}$ Apply the logarithm laws
$\log 1 = \log 2 - 0.234t\log(1.3)$
$\log 1 - \log 2 = -0.234t\log(1.3)$ Isolate the variable
$\frac{\log1-\log2}{-0.234\log1.3} = t$
$t = 11.2903$ Solve with your calculator

Make sure you use the proper parenthetical formation.

It will take approximately 11.3 years Answer the question.

Earthquakes, Sound, pH

Logarithms can be used to solve applications comparing the intensity of earthquakes, the intensity of sounds, and the acidity or alkalinity of solutions. The Richter scale for earthquakes, the decibel scale for sounds and the pH scale for solutions are all base 10.

$$I = I_0(10)^{R-r}$$

where:

I = intensity of a stronger earthquake
I_o = intensity of weaker earthquake

R = Richter magnitude of stronger earthquake
r = Richter magnitude of weaker earthquake

$$I = I_0(10)^{(D-d)/10}$$

where:

I = intensity of a louder sound
I_o = intensity of a softer sound

D = decibel level of louder sound
d = decibel level of softer sound

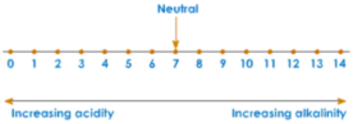
$$I = I_0(10)^{P-p}$$

where:

I = new solution that is compared to original one
I_o = "original" solution

P = larger pH reading
p = smaller pH reading

- A neutral solution has a pH of 7.
- Solutions with pH larger than 7 are basic, or alkaline.
- Solutions with pH smaller than 7 are acidic.



$pH = -\log\left[H^+\right]$ where $\left[H^+\right]$ is the hydrogen ion concentration in moles per liter

The exponent is always a difference: larger reading – smaller reading
For sound questions, divide each decibel reading by 10.

Example

1a) In 1983, an earthquake measuring 5.5 on the Richter scale occurred in Columbia. In 1989, the San Francisco earthquake measured 6.9 on the Richter scale. How much more intense was the San Francisco earthquake than the Columbia earthquake?

r R

$I = I_0(10)^{R-r}$ $I_0 = ①$ ∴ can ignore I_0

$I = 10^{6.9-5.5} \rightarrow I = 10^{1.4}$ calculator $\rightarrow$ $\boxed{I = 25.12 \text{ times more intense}}$

S.F earthquake is

than Columbia's earthquake

b) Calculate the magnitude of an earthquake that is 1500 times as intense as the Columbia earthquake.

$I = \cancel{I_0}(10)^{R-r} \rightarrow I = 10^{R-r} \longrightarrow \log 1500 = \log 10^{(R-5.5)}$

$\overset{1}{}$ I

$\log 1500 = (R-5.5)\cancel{\log 10} \rightarrow \log 1500 = R-5.5 \rightarrow \log 1500 + 5.5 = R$

=1 +5.5

$\boxed{R = 8.7}$

8.68

2. How much louder is a sound with an intensity of 112 dB compared to a sound with an intensity of 90 dB

$I = 10^{\frac{D-d}{10}} \rightarrow I = 10^{\frac{112-90}{10}} \rightarrow I = 10^{\frac{22}{10}} \rightarrow I = 10^{2.2}$

$I = 158.49$ times louder

b) If three different jets are flying together at an air show, each with a sound level of 120 decibels, then find the approximate total decibel level.

3 times the Intensity of one jet ∴ $I = 3$

$I = 10^{\frac{D-d}{10}}$

$\log 3 = \log 10^{\frac{D-120}{10}}$

$\log 3 = \frac{D-120}{10}\cancel{\log 10}$

$(10)\log 3 = \frac{D-120}{\cancel{10}}\cancel{(10)}$

$10\log 3 = D-120$

+120

$10\log 3 + 120 = D$

$\boxed{D = 124.78 \text{ total decibels of 3 planes.}}$

Example pH $= -\log\left[H^+\right]$ same $I = I_0(10)^{P-p}$

a) A beaker of acid has a hydrogen ion concentration of 3.5×10^{-6} mol/L. Calculate the pH of the acid.

$pH = -\log(3.5\times10^{-6})$ } $\Rightarrow \boxed{pH = 5.5}$ acidic

Sci. Notation.

or $pH = -\log(0.0000035)$

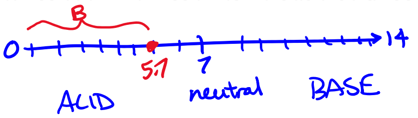
3.5×10^{-6} ← 6 decimal places left

$= 0.0000035$

ex 3.5×10^{6} ← 6 decimals to right

$= 3500000$

b) Solution A has a pH of 5.7. Solution B is 1260 times more **acidic** than Solution A. Find the pH of Solution B.



$I = 10^{P-p}$

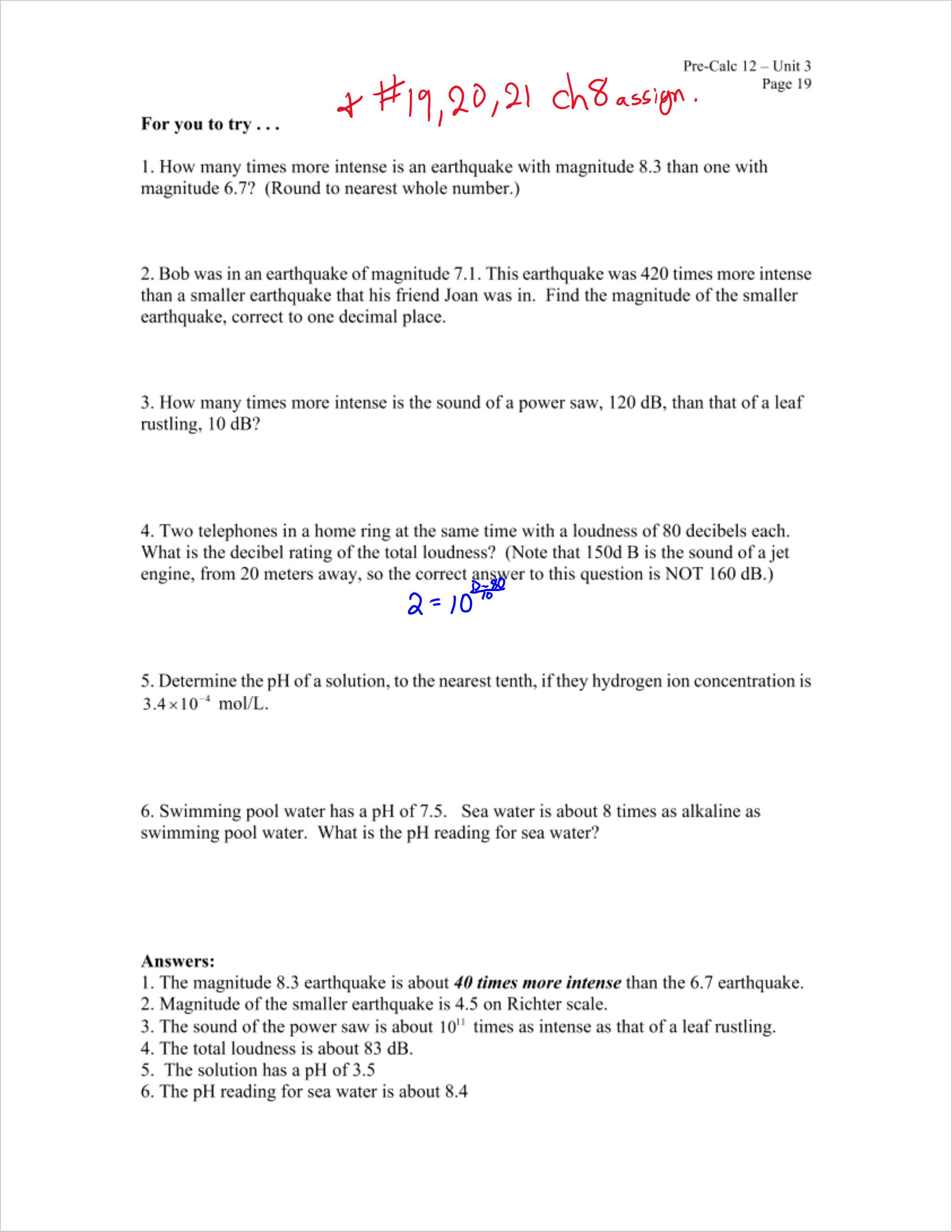
$\log 1260 = \log 10^{5.7-p}$

$\log 1260 = (5.7-p)\cancel{\log 10}$

$\log 1260 = 5.7 - p$

+p −log1260

$p = 5.7 - \log 1260 \rightarrow \boxed{pH = 2.6}$



+ #19, 20, 21 ch8 assign.

For you to try . . .

1. How many times more intense is an earthquake with magnitude 8.3 than one with magnitude 6.7? (Round to nearest whole number.)

2. Bob was in an earthquake of magnitude 7.1. This earthquake was 420 times more intense than a smaller earthquake that his friend Joan was in. Find the magnitude of the smaller earthquake, correct to one decimal place.

3. How many times more intense is the sound of a power saw, 120 dB, than that of a leaf rustling, 10 dB?

4. Two telephones in a home ring at the same time with a loudness of 80 decibels each. What is the decibel rating of the total loudness? (Note that 150d B is the sound of a jet engine, from 20 meters away, so the correct answer to this question is NOT 160 dB.)

$2 = 10^{\frac{D-80}{10}}$

5. Determine the pH of a solution, to the nearest tenth, if they hydrogen ion concentration is 3.4×10^{-4} mol/L.

6. Swimming pool water has a pH of 7.5. Sea water is about 8 times as alkaline as swimming pool water. What is the pH reading for sea water?

Answers:

1. The magnitude 8.3 earthquake is about ***40 times more intense*** than the 6.7 earthquake.
2. Magnitude of the smaller earthquake is 4.5 on Richter scale.
3. The sound of the power saw is about 10^{11} times as intense as that of a leaf rustling.
4. The total loudness is about 83 dB.
5. The solution has a pH of 3.5
6. The pH reading for sea water is about 8.4

$A = A_0(b)^{t/p}$

growth/decay

ex: doubling $b=2$
half-life $b=\frac{1}{2}$ or $b=0.5$

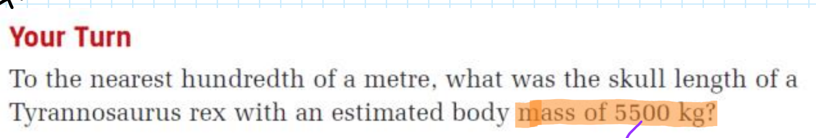
$A = P(1+i)^n$

$i = \frac{r}{n}$ $n = tn$

compounding period.

ex: monthly = $n=12$
weekly = $n=52$

p.410



Your Turn

To the nearest hundredth of a metre, what was the skull length of a Tyrannosaurus rex with an estimated body mass of 5500 kg?

formula ⇒ $3.6022 \log S = \log m - 3.4444$ (given)

$$\frac{3.6022 \log S}{3.6022} = \frac{\log 5500 - 3.4444}{3.6022}$$

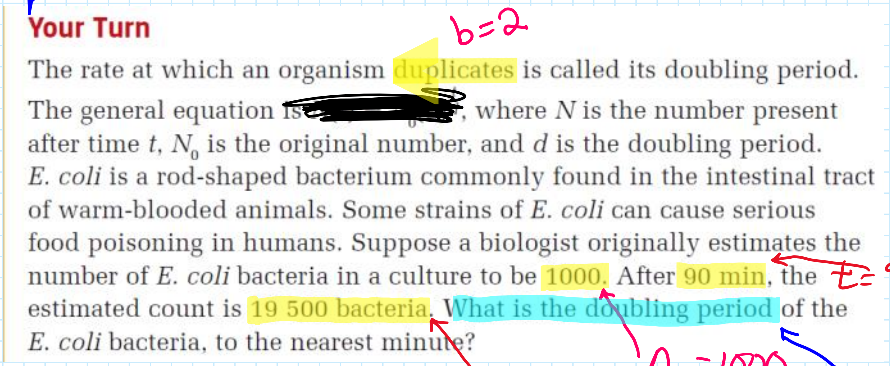
$$\log_{10} S = \frac{\log 5500 - 3.4444}{3.6022}.$$

BOOT

$$S = 10^{\frac{\log 5500 - 3.4449}{3.6022}}$$

$S = 1.2$ m length of skull.

p.411



Your Turn

The rate at which an organism duplicates is called its doubling period. The general equation is [redacted], where N is the number present after time t, N_0 is the original number, and d is the doubling period. *E. coli* is a rod-shaped bacterium commonly found in the intestinal tract of warm-blooded animals. Some strains of *E. coli* can cause serious food poisoning in humans. Suppose a biologist originally estimates the number of *E. coli* bacteria in a culture to be 1000. After 90 min, the estimated count is 19 500 bacteria. What is the doubling period of the *E. coli* bacteria, to the nearest minute?

$b=2$

$t = 90$min

$A = 19500$

$A_0 = 1000$

p ?

$$A = A_0 (b)^{t/p}$$

$$\frac{19500}{1000} = \frac{1000 (2)^{\frac{90}{p}}}{1000}$$

leave exact

$$\log 19.5 = \log 2^{\frac{90}{p}}$$

$$(p) \log 19.5 = \frac{90}{p} \log 2 \; (p)$$

$$\frac{p \log 19.5}{\log 19.5} = \frac{90 \log 2}{\log 19.5}$$

$$p = \frac{90 \log 2}{\log 19.5}$$

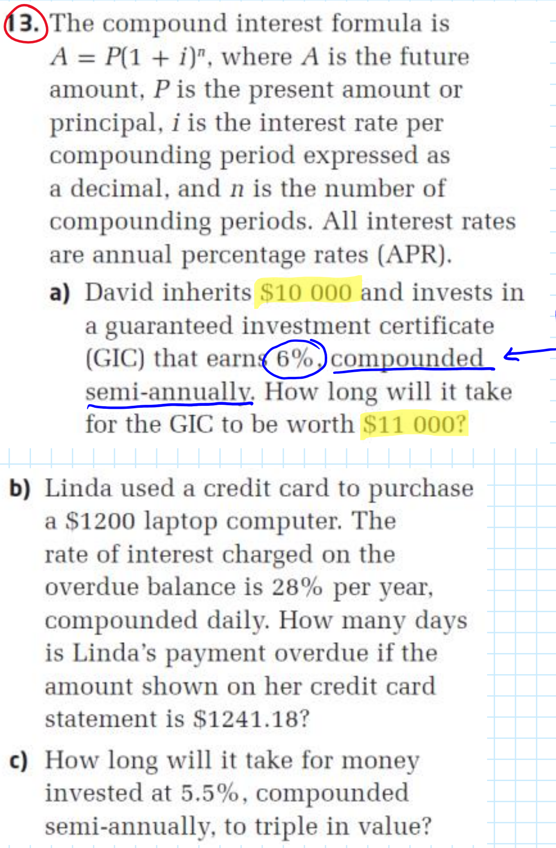
$$\boxed{p = 21.0 \text{ min is doubling period}}$$

① Determine formula

② ÷ by initial amount

③ log both side.
↳ power law

④ solve



⑬. The compound interest formula is $A = P(1 + i)^n$, where A is the future amount, P is the present amount or principal, i is the interest rate per compounding period expressed as a decimal, and n is the number of compounding periods. All interest rates are annual percentage rates (APR).

a) David inherits \$10 000 and invests in a guaranteed investment certificate (GIC) that earns 6%, compounded semi-annually. How long will it take for the GIC to be worth \$11 000?

b) Linda used a credit card to purchase a \$1200 laptop computer. The rate of interest charged on the overdue balance is 28% per year, compounded daily. How many days is Linda's payment overdue if the amount shown on her credit card statement is \$1241.18?

c) How long will it take for money invested at 5.5%, compounded semi-annually, to triple in value?

p.414

$i = \frac{r}{n}$ $\quad$ $n = tn$

$r = 0.06$

$n = 2$

$i = \frac{0.06}{2}$ $\quad$ $n = t(2)$

$n = 2t$

formula

$$A = P(1+i)^n$$

$$\frac{11000}{10000} = \frac{10000 (1 + 0.03)^{2t}}{10000}$$

$$\log 1.1 = \log (1.03)^{2t}$$

$$\frac{\log 1.1}{2 \log 1.03} = \frac{2t \log 1.03}{2 \log 1.03}$$

$$t = \frac{\log 1.1}{(2 \log 1.03)}$$

$t = 1.6$ yr $\quad$ (not 2.66×10^{-4})

log(1.1)/2 log1.03

invested at 5.5%, compounded semi-annually, to triple in value?

$t = 1.6$ yr (not 2.66×10^{-4})

$\log(1.1)/2$ $\log 1.03$

15. Swedish researchers report that they have discovered the world's oldest living tree. The spruce tree's roots were radiocarbon dated and found to have 31.5% of their carbon-14 (C-14) left. The half-life of C-14 is 5730 years. How old was the tree when it was discovered?

16. Radioisotopes are used to diagnose various illnesses. Iodine-131 (I-131) is administered to a patient to diagnose thyroid gland activity. The original dosage contains 280 MBq of I-131. If none is lost from the body, then after 6 h there are 274 MBq of I-131 in the patient's thyroid. What is the half-life of I-131, to the nearest day?

a. Carbon-14 has a half-life of 5740 years. How long would it take a 5.3 gram sample to decay to 1.9 grams.

b. A certain kind of mold triples every 7 days. How long would it take a sample of 125 cells to grow to 100,000 cells?

c. The population of Langley increases by 0.5% per year with a current population of 127,000 people. What would the populations be in 10 years?

How long would it take for the populations to reach 200,000 people?

d. If an earthquake in Town A is 3500 times more intense than an earthquake of magnitude 4.2 in Town B, what is the magnitude of the earthquake in Town A?

e. If the pH of stomach acid at 2.2 is 50,000 times more acidic than urine, what is the pH of urine?

13. The following function represents the blood pressure, in mmHg, of young adults related to the blood vessel volume, in microlitres.

$$V(p) = 0.23 + 0.35\log(p - 56.1)$$

a. Determine the blood vessel volume if the blood pressure is 120mmHg.

b. Determine the blood pressure if the vessel volume is 0.9 microlitres.

8.4 Practice

Extra Practice for Chapter 8.4

Applications

3. A scientist started with a culture of 20 bacteria in a dish. He noticed that after 80 hours, there were 1800 bacteria. What is the doubling time of this bacteria?

4. At the beginning of the year, you deposit $1000 into a bank account, with an annual interest rate of 5%. Assume no other deposits or withdrawals are made and the interest rate stays constant.

a) what will be the value of the account after 5 years if interest is compounded annually?

b) how long will it be when his money doubles in value?

Extra Practice for Chapter 8.4

5. When people take a particular medicine, the drug is metabolised and eliminated at a certain rate. Suppose the initial amount of a drug in the body is 200 mg and is eliminated at a rate of 30% per hour. How long will it take to reach 10 mg?

6. Certain bacteria, given favourable growth conditions, grow continuously at a rate of 4.6% a day. Find the bacterial population after thirty-six hours, if the initial population was 250 bacteria.

Extra Practice for Chapter 8.4

7. A penicillin solution has a half-life of 6 days. How long will it take for the concentration to drop to 70% of the initial concentration?

8. What is the magnitude of the earthquake in City A if the earthquake in City B has a magnitude of 5.7 on the Richter scale and is 4500 times as intense?

Extra Practice for Chapter 8.4

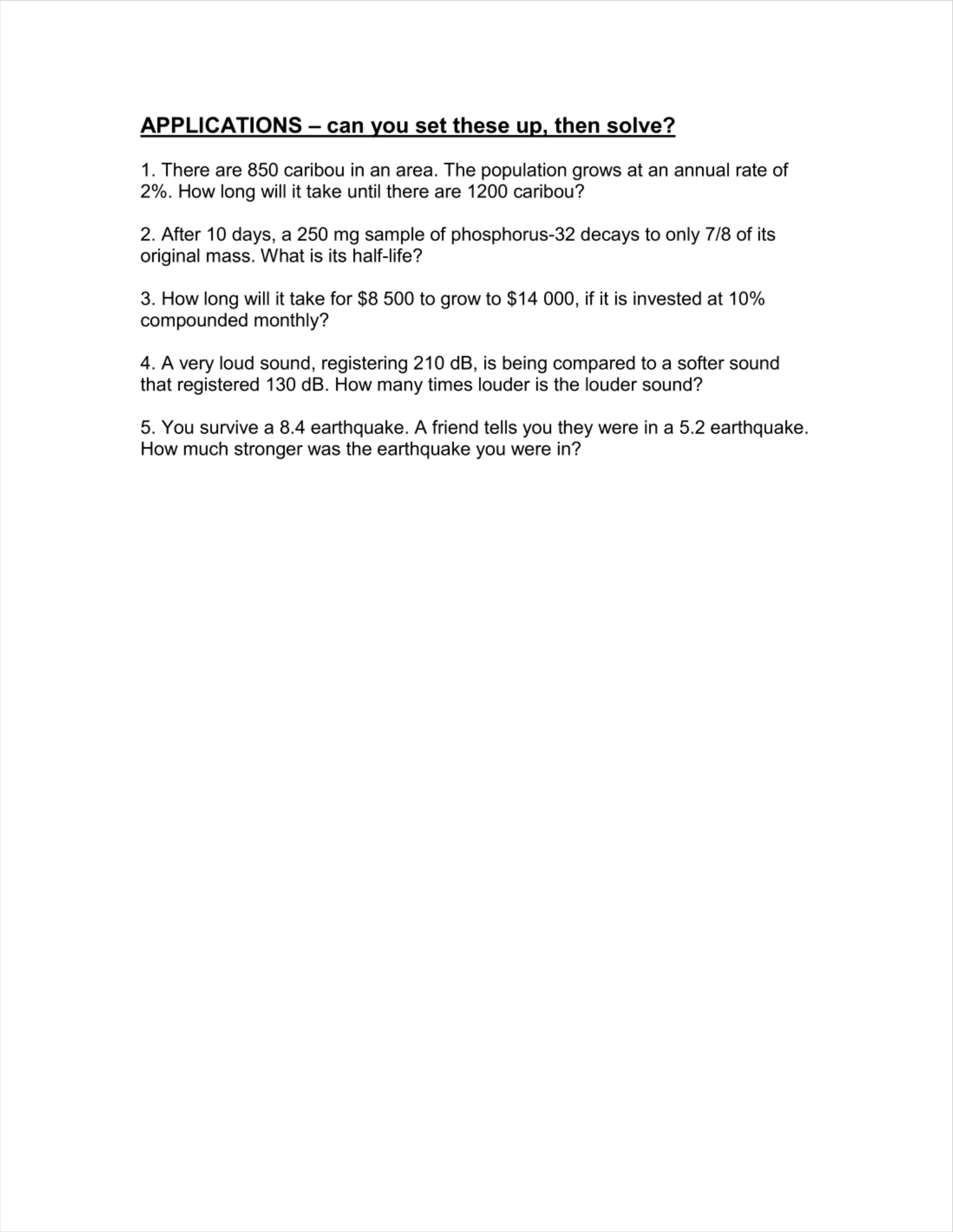
9. What is the pH of a tomato if it is 15000 times more acidic than hand soap with a pH of 9.5?

10. It is said that the eardrum can rupture at a decibel level that is 100,000,000 times as intense as the normal sound level of a vacuum at 70Db on the Decibel scale (that would be like listening to a jet at take-off). At what Db value on the scale can the eardrum rupture?

C_21 Key & Applications Set-up Solve

6:12 PM

C_21 Setting Up Equations and Key



APPLICATIONS – can you set these up, then solve?

1. There are 850 caribou in an area. The population grows at an annual rate of 2%. How long will it take until there are 1200 caribou?

2. After 10 days, a 250 mg sample of phosphorus-32 decays to only 7/8 of its original mass. What is its half-life?

3. How long will it take for $8 500 to grow to $14 000, if it is invested at 10% compounded monthly?

4. A very loud sound, registering 210 dB, is being compared to a softer sound that registered 130 dB. How many times louder is the louder sound?

5. You survive a 8.4 earthquake. A friend tells you they were in a 5.2 earthquake. How much stronger was the earthquake you were in?

SOLUTIONS

1. There are 850 caribou in an area. The population grows at an annual rate of 2%. How long will it take until there are 1200 caribou?

$$\frac{1200}{850} = \frac{850}{850}(1.02)^n$$

$$\frac{1200}{850} = 1.02^n$$

$$\log\left(\frac{1200}{850}\right) = \log 1.02^n$$

$$\log\left(\frac{1200}{850}\right) = n\log 1.02$$

$$\frac{\log(1200/850)}{\log 1.02} = n, \quad \boxed{n = 17.4 \text{ years}}$$

2. After 10 days, a 250 mg sample of phosphorus-32 decays to only 7/8 of its original mass. What is its half-life?

$$\frac{\frac{7}{8}(25\cancel{0})}{\cancel{2}50} = \frac{25\cancel{0}(0.5)^{10/h}}{\cancel{2}50}$$

$$\frac{7}{8} = (0.5)^{10/h}$$

$$\log(7/8) = \log(0.5)^{10/h}$$

$$h \times \left[\log 7/8\right] = \left[\frac{10}{h}(\log 0.5)\right] \times \cancel{h}$$

$$h = \frac{10\log 0.5}{\log(7/8)}$$

$$\boxed{h = 51.9 \text{ days}}$$

3. How long will it take for $8 500 to grow to $14 000, if it is invested at 10% compounded monthly?

$$\frac{14,00\cancel{0}}{85\cancel{0}} = \frac{8500}{\cancel{8}500}\left(1 + \frac{0.10}{12}\right)^n$$

$$\frac{140}{85} = (1.008\overline{3})^n$$

$$\log\left(\frac{140}{85}\right) = (\log 1.008\overline{3})^n$$

$$\log\left(\frac{140}{85}\right) = n\log 1.008\overline{3}$$

$$n = \frac{\log\left(\frac{140}{85}\right)}{\log 1.008\overline{3}}$$

$n = 60.128$ compounding periods

$$\Rightarrow \frac{60.128}{12} = \boxed{5.01 \text{ years}}$$

4. A very loud sound, registering 210 dB, is being compared to a softer sound that registered 130 dB. How many times louder is the louder sound?

$$I = I_0(10)^{\frac{D-d}{10}}$$

$$I = I_0(10)^{\frac{210-130}{10}}$$

$$I = I_0(10)^{80/10}$$

$$I = I_0(10)^{8}$$

It's 10^8 times as loud.

5. You survive a 8.4 earthquake. A friend tells you they were in a 5.2 earthquake. How much stronger was the earthquake you were in?

$$I = I_0(10)^{R-r}$$

$$I = I_0(10)^{8.4-5.2}$$

$$I = I_0(10)^{3.2}$$

$$I \doteq I_0(1584.89)$$

Your earthquake is about 1584.89 times as strong as your friend's.