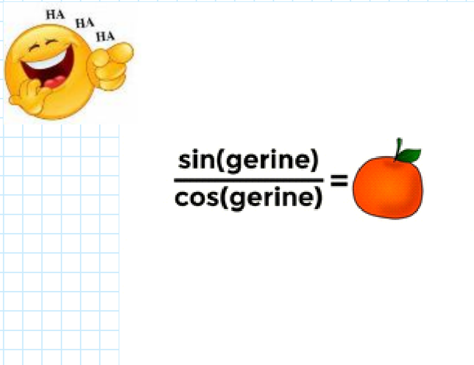
Class_20 Nov 17 - Log Equations and Applications

Sunday, November 6, 2022 8:42 PM

Tonight's Class:

- **8.3 Logarithm Laws**
- **8.4 Log Equations and Applications**
- **Unit 3 Test (Chapters 7 and 8) on Thursday, Nov 24**



Last time we looked at two kinds of questions involving logs:

$$\log_2 2 + \log_2 16 = 1 + 4 = 5 \qquad \log_2(32) = 5$$

$$\log_5 125 - \log_5 25 = 3 - 2 = 1 \qquad \log_5(5) = 1$$

8.3 Laws of Logarithms

product

Product Law:	$\log_c(MN) = \log_c M + \log_c N$
Quotient Law:	$\log_c\left(\frac{M}{N}\right) = \log_c M - \log_c N$
Power Law:	$\log_c(M^P) = P\log_c M$

$\log(35x^2y^3) = \log(35) + \log(x^2) + \log(y^3)$

To Try:

1. Evaluate without using the "log" button: $\log_3 54 - \log_3 2 = \log_3\left(\frac{54}{2}\right)$ $= \log_3(27) = 3$

2. Find the value of each of the following without using a calculator:

   a) $\ln 1 = \log_e 1 = 0$ $e^0 = 1$

   b) $\ln e = \log_e e = 1$

   c) $\ln e^4 = 4 \cdot \ln e = 4 \cdot 1 = 4$

   $\ln e^4 = \log_e e^4 = 4$

3. Evaluate without using the "log" button: $\log_{14} 4 + \log_{14} 49 = \log_{14}(4 \cdot 49)$ $= \log_{14}(196)$ $= \log_{14}(14^2)$ $= 2\log_{14} 14$ $= 2 \cdot 1$ $= 2$

Change of Base Formula: $\log_C A = \frac{\log_B A}{\log_B C}$

$\log_c A = x$

Changed form: $c^x = A$

log both sides: $\log c^x = \log A$

Power Law: $x \log C = \log A$, $\frac{x\log C}{\log C} = \frac{\log A}{\log C}$

Solve for x: $x = \frac{\log A}{\log C}$

1. Evaluate. Give answer correct to 4 decimal places. $\log_2 18 = \frac{\log 18}{\log 2} \doteq 4.1699$

2. Express as a single logarithm. $\frac{\log 30}{\log 5} = \log_5 30$

3. Rewrite this equation so you can graph it on a graphing calculator: $y = \log_4 x$

   $y = \frac{\log x}{\log 4}$

For the test – graphing calculator / Desmos scientific

Know these 4 log laws.

1) Product Law: $\log_c(XY) = \log_c X + \log_c Y$

2) Quotient Law: $\log_c\left(\frac{P}{Q}\right) = \log_c P - \log_c Q$

3) Power Law: $\log_c W^T = T\log_c W$

4) Change of Base Law: $\log_K Z = \frac{\log_c Z}{\log_c K}$

expand or condense

1. $\log_3(4y^2)$ $= \log_3 4 + \log_3 y^2$ product law, power law $= \log_3 4 + 2\log_3 y$	6. $\frac{1}{2}(\log b - \log c) = \frac{1}{2}\log b - \frac{1}{2}\log c$ $= \log b^{1/2} - \log c^{1/2}$ $= \log\left(\frac{b^{1/2}}{c^{1/2}}\right)$ $= \log\left(\frac{\sqrt{b}}{\sqrt{c}}\right)$ OR $\log\left(\sqrt{\frac{b}{c}}\right)$
2. $2\log_4 b + 3\log_4 c = \log_4 b^2 + \log_4 c^3$ $= \log_4(b^2c^3)$	7. $\log\left(\frac{\sqrt{a}}{c^2}\right) = \log\left(\frac{a^{1/2}}{c^2}\right)$ $= \log a^{1/2} - \log c^2$ $= \frac{1}{2}\log a - 2\log c$
3. $\ln(ab) = \ln(a) + \ln(b)$ $\log_e a + \log_e b$	8. $\log(x^2y)^4 = 4\log(x^2y)$ — power law $= 4[\log x^2 + \log y]$ — product law power law $= 4[2\log x + \log y]$ $= 8\log x + 4\log y$
4. $\log\left(\frac{a}{b}\right) = \log a - \log b$	9. $3\log x - \log w^2$ $= \log x^3 - \log w^2$ $= \log\left(\frac{x^3}{w^2}\right)$
5. $\frac{1}{2}\log a + 2\log c$ $= \log a^{1/2} + \log c^2$ $= \log(a^{1/2}c^2)$ $= \log(\sqrt{a}\,c^2)$	10. $\log\left(\frac{1000a^2}{c}\right) = \log(1000a^2) - \log c$ $= \log_{10}1000 + \log a^2 - \log c$ $= 3 + 2\log a - \log c$

OR
$\frac{1}{2}(\log b - \log c)$
$= \frac{1}{2}\left(\log\left(\frac{b}{c}\right)\right)$
$= \log\left(\frac{b}{c}\right)^{1/2}$
$= \log\sqrt{\frac{b}{c}}$

11. $\log_5(5x\sqrt{y}) = \log_5 5 + \log_5 x + \log_5\sqrt{y}$ $= 1 + \log_5 x + \log_5 y^{1/2}$ $= 1 + \log_5 x + \frac{1}{2}\log_5 y$	16. $\log_7 y - 2\log_7 w + \log_7(5x)$ $= \log_7 y - \log_7 w^2 + \log_7(5x)$ $= \log_7\left(\frac{y}{w^2}\right) + \log_7(5x)$ $= \log_7\left(\frac{y}{w^2}\cdot 5x\right) = \log_7\left(\frac{5xy}{w^2}\right)$
12. $\log\left(\frac{\sqrt{bc}}{a}\right) = \log(bc)^{1/2} - \log a$ $= \frac{1}{2}\log(bc) - \log a$ $= \frac{1}{2}[\log b + \log c] - \log a$ $= \frac{1}{2}\log b + \frac{1}{2}\log c - \log a$	17. $\log a + 3\log b - 2\log c$ $= \log a + \log b^3 - \log c^2$ $= \log(ab^3) - \log c^2$ $= \log\left(\frac{ab^3}{c^2}\right)$
13. $2\log a - 4\log b$ $= \log a^2 - \log b^4$ $= \log\left(\frac{a^2}{b^4}\right)$	18. $5\log_4 2 - \frac{1}{3}\log_4 8$ $= \log_4 2^5 - \log_4 8^{1/3}$ $= \log_4\left(\frac{2^5}{8^{1/3}}\right) = \log_4\left(\frac{32}{2}\right) = \log_4(16)$ $= 2$
14. $\log(a^2c) = \log a^2 + \log c$ $= 2\log a + \log c$	19. $\frac{\log_5 x}{4} - \log_5(3x)$ $= \frac{1}{4}\log_5 x - \log_5 3x$ $= \log_5 x^{1/4} - \log_5 3x$ $= \log_5\left(\frac{\sqrt[4]{x}}{3x}\right)$
15. $\log\left(\frac{x}{yw}\right) = \log x - \log(yw)$ $= \log x - [\log y + \log w]$ $= \log x - \log y - \log w$	20. $2\log c - (3\log a + \log b)$ $= \log c^2 - (\log a^3 + \log b)$ $= \log c^2 - (\log(a^3b))$ $= \log\left(\frac{c^2}{a^3b}\right)$

OR $\log_5\left(\frac{x^{1/4}}{3x}\right)$
$= \log_5\left(\frac{x^{1/4 - x}}{3}\right)$
$= \log_5\left(\frac{x^{-3/4}}{3}\right)$
$= \log_5\left(\frac{1}{3x^{3/4}}\right)$

Know these 4 log laws.

1) Product Law: $\log(AB) = \log(A) + \log(B)$
2) Quotient Law: $\log\left(\frac{X}{Y}\right) = \log X - \log Y$
3) Power Law: $\log(A^P) = P\log A$
4) Change of Base Law:

$$\log_c A = \frac{\log A}{\log C}$$

Small WB - Using Log Laws

TB p 396

Your Turn

Use the laws of logarithms to simplify and evaluate each expression.

b) $\log_5 1000 - \log_5 4 - \log_5 2$ $= \log_5\left(\frac{1000}{4\cdot 2}\right)$

$= \log_5 125$

$= 3$

OR

$= \log_5\left(\frac{1000}{4}\right) - \log_5 2$

$= \log_5(250) - \log_5 2$

$= \log_5\left(\frac{250}{2}\right) = \log_5(125) = 3$

c) $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

$= \log_3 6^2 - \log_3 64^{1/2} + \log_3 2$

$= \log_3\left(\frac{36\cdot 2}{64^{1/2}}\right)$

$= \log_3\left(\frac{72}{\sqrt{64}}\right) = \log_3\left(\frac{72}{8}\right) = \log_3 9 = 2$

TB p 401

8. If $\log 3 = P$ and $\log 5 = Q$, write an algebraic expression in terms of P and Q for each of the following.

a) $\log\left(\frac{3}{5}\right)$ $= \log 3 - \log 5 = P - Q$

b) $\log 15$ $= \log(3\times 5) = \log 3 + \log 5 = P + Q$

c) $\log 3\sqrt{5}$ $= \log(3) + \log\sqrt{5} = P + \log 5^{1/2}$

$= P + \frac{1}{2}\log 5$

$= P + \frac{1}{2}Q$

d) $\log \frac{25}{9}$

$\log\left(\frac{5^2}{3^2}\right) = \log 5^2 - \log 3^2 = 2\log 5 - 2\log 3$

$= 2P - 2Q$

8.4 Logarithmic and Exponential Equations

Solving Logarithmic Equations

1. Use logarithm laws to simplify equation into one of two forms:
   - $\log_c(\text{argument}) = \text{number}$
     - in this case, change to exponential form and solve
   - $\log_c(\text{argument}) = \log_c(\text{another argument})$
     - in this case, set the two arguments equal

2. Use algebra to solve the equation you created in step 1.

3. Substitute each solution into the original equation. If the solution makes the argument become zero or a negative number, then it is an ***extraneous solution*** and must be rejected.

To Try: Round to two decimal places, if necessary.
Solve for x. Reject any extraneous solutions.

1. $\log_9 5 + \log_9 x = \log_9 30$

product laws $\log_9(5x) = \log_9 30$

$\Rightarrow \frac{5x}{5} = \frac{30}{5}$ $\boxed{x=6}$

OR
$\log_9(5x) - \log_9(30) = 0$
$\log_9\left(\frac{5x}{30}\right) = 0$
$9^0 = \frac{5x}{30}$
$30 \cdot 1 = \frac{5x}{30} \cdot 30$
$\frac{30}{5} = \frac{5x}{5}$ $\boxed{x=6}$

2. $\ln x + \ln 5 = 2$

$\ln_e(5x) = 2$
change to exponential form
$\frac{e^2}{5} = \frac{5x}{5}$

exact $\boxed{x = \frac{e^2}{5}}$ OR rounded $\boxed{x \doteq 1.48}$

3. $\ln 512 - \ln 8 = 3\ln x$

$\ln\left(\frac{512}{8}\right) = \ln(x^3)$
$\ln(64) = \ln(x^3)$
$\Rightarrow (64)^{1/3} = (x^3)^{1/3}$ $\boxed{x=4}$

4. $\log_2(x-6) = 3 - \log_2(x-4)$

$\log_2(x-6) + \log_2(x-4) = 3$
$\log_2[(x-6)(x-4)] = 3$
$\log_2(x^2 - 4x - 6x + 24) = 3$
$\log_2(x^2 - 10x + 24) = 3$

Change to exp. form:
$2^3 = x^2 - 10x + 24$
$8 = x^2 - 10x + 24$ (-8, -8)
$0 = x^2 - 10x + 16$
$0 = (x-2)(x-8)$
$\cancel{x=2}$ extraneous root (bogus answer) $\boxed{x=8}$

5. $2\log_4(x+4) - \log_4(x+12) = 1$

$\log_4\left[\frac{(x+4)^2}{x+12}\right] = 1$
$\log_4\left(\frac{(x+4)(x+4)}{x+12}\right) = 1$
$\log_4\left(\frac{x^2+8x+16}{x+12}\right) = 1$

Change form: $(x+12)\,4 = \left(\frac{x^2+8x+16}{\cancel{x+12}}\right)\cancel{(x+12)}$
$4x + 48 = x^2 + 8x + 16$ ($-4x$, -48)
$0 = x^2 + 4x - 32$
$0 = (x-4)(x+8)$
$\boxed{x=4}$ $\cancel{x=-8}$ extraneous root
reject it, it makes an argument negative.

6. $\log_{12}(3-x) + \log_{12}(2-x) = 1$

$\log_{12}(3-x)(2-x) = 1$
$\log_{12}(6 - 3x - 2x + x^2) = 1$
$\log_{12}(6 - 5x + x^2) = 1$
$12^1 = x^2 - 5x + 6$ (-12, -12)
$0 = x^2 - 5x - 6$
$0 = (x-6)(x+1)$
$\cancel{x=6}$ extraneous $\boxed{x=-1}$

Solving Exponential Equations with Different Bases

In chapter 7 we solved exponential equations by making each side of the equation use the same base. When that is hard to do, we can instead solve by taking the logarithm of each side of the original equation and solving the resulting equation.

Solving Exponential Equations with Different Bases

In chapter 7 we solved exponential equations by making each side of the equation use the same base. When that is hard to do, we can instead solve by taking the logarithm of each side of the original equation and solving the resulting equation.

To Try:
Solve for x. **Solve correct to 2 decimal places.**

1. $3^x = 2800$

$$\log 3^x = \log 2800$$
$$\frac{x \cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 2800}{\log 3}$$
$$\boxed{x = \frac{\log 2800}{\log 3}}$$ exact form

$$\boxed{x \doteq 7.22}$$

2. $e^x = 2$

$$\log e^x = \log 2$$
$$\frac{x \cancel{\log e}}{\cancel{\log e}} = \frac{\log 2}{\log e}$$
$$x = \frac{\log 2}{\log e}$$
$$\boxed{x \doteq 0.69}$$

3. $3\left(4^{2x+3}\right) = 8^{4x-2}$

$$\log\left[3\left(4^{2x+3}\right)\right] = \log 8^{4x-2}$$

this argument is a PRODUCT

$$\log 3 + \log 4^{2x+3} = \log 8^{4x-2}$$
$$\log 3 + (2x+3)\log 4 = (4x-2)\log 8$$ (power law)
$$\log 3 + 2x\log 4 + 3\log 4 = 4x\log 8 - 2\log 8$$ (distribute)
$$\log 3 + 3\log 4 + 2\log 8 = 4x\log 8 - 2x\log 4$$ (collect x-terms)
$$\frac{(\log 3 + 3\log 4 + 2\log 8)}{(4\log 8 - 2\log 4)} = \frac{x\cancel{(4\log 8 - 2\log 4)}}{\cancel{(4\log 8 - 2\log 4)}}$$

$$\boxed{x \doteq 1.70}$$

For next class, Tuesday, Nov 22

- **Complete Chapter 8 Hand-in (#1-15 for now)**
- **Prepare for the Unit 3 Test (Chapters 7-8, including "e" and natural log) It will be on THURSDAY, Nov 24.**
- **Can use scientific calculator, graphing calculator, and/or Desmos SCIENTIFIC calculator on this exam.**

Study Suggestions:
- **Complete optional worksheets (posted on website):**
  - **More Solving Practice (Log & Exponential Equations)**
  - **Applications Set-up & Solving**
  - **Chapter 8 Review**
  - **Unit 3 Practice Test**

- **Equation solving:**

- **TB p 412: 1, 2ac, 3, 4ac, 5, 6, 7acd, 8abd, 13, 16**

Log Scale questions:

- **TB p 401: 13bc, 16bc**
- **TB p 417: 15, 17**
- **TB p 419: 6, 15**