

Tonight's Class:

- Questions from 6.2-6.3?
- Working through sections 6.4-6.5
 - Adding/Subtraction Rational Expressions (continued)
 - Solving Rational Equations
- Work on practice questions from worktext

Page 560 - part (b)

Example 1 Adding and Subtracting Rational Expressions with Binomial Denominators

Simplify:

a) $\frac{4n}{n+4} + \frac{3n}{n-5}$

b) $\frac{1}{x^2-36} - \frac{1}{6x-x^2}$

b) $\frac{1}{x^2-36} - \frac{1}{6x-x^2}$

1) Factor

2) NPVs:
 $x \neq -6$
 $x \neq 6$
 $x \neq 0$

LCD = product of every factor that appears in the denominators.
 $x(x+6)(x-6)$

$$= \frac{1}{(x+6)(x-6)} - \frac{1}{x(6-x)}$$

$$= \frac{1}{(x+6)(x-6)} - \frac{1}{x(-6+x)}$$

$$= \frac{1}{(x+6)(x-6)} + \frac{1}{x(x-6)}$$

$$= \frac{1 \cdot x}{(x+6)(x-6)x} + \frac{1 \cdot (x+6)}{x(x-6)(x+6)}$$

$$= \frac{x + x + 6}{x(x+6)(x-6)}$$

$$= \frac{2x + 6}{x(x+6)(x-6)}$$

$$= \frac{2(x+3)}{x(x+6)(x-6)}$$

LCM = 20

$$\frac{2}{5} + \frac{1}{4}$$

$$\frac{2 \cdot 4}{5 \cdot 4} + \frac{1 \cdot 5}{4 \cdot 5}$$

$$= \frac{8 + 5}{20} = \frac{13}{20}$$

Page 561 - use for foldable on subtraction (and addition, if you didn't already fill that one in)

Example 2 Adding and Subtracting with Binomial and Trinomial Denominators

Simplify:

a) $\frac{5}{x^2-25} + \frac{4}{x^2+10x+25}$

b) $\frac{n-2}{n^2-5n+6} - \frac{n+4}{n^2-11n+30}$

a) $\frac{5}{x^2-25} + \frac{4}{x^2+10x+25}$

mult. = 25
 add = 10
 5, 5

$\frac{5}{(x+5)(x-5)} + \frac{4}{(x+5)(x+5)}$

NPVs: $x \neq -5, x \neq 5$

(least common denominator) → LCD: $(x-5)(x+5)(x+5)$

$\frac{5 \cdot (x+5)}{(x+5)(x-5)(x+5)} + \frac{4 \cdot (x-5)}{(x+5)(x+5)(x-5)}$

$\frac{(x+5)(x-5)}{(x+5)(x-5)(x+5)}$
 $\frac{(x+5)(x+5)}{(x+5)(x-5)(x+5)}$

LCM) $(x+5)(x-5)(x+5)$

Common denominator) \rightarrow LCD: $(x-5)(x+5)(x+5)$

LCD) $(x+5)(x-5)(x+5)$

$$\frac{5}{(x+5)(x-5)} \cdot \frac{(x+5)}{(x+5)} + \frac{4}{(x+5)(x+5)} \cdot \frac{(x-5)}{(x-5)}$$

$$\frac{5(x+5) + 4(x-5)}{(x+5)(x-5)(x+5)}$$

$$\frac{5x + 25 + 4x - 20}{(x+5)(x-5)(x+5)}$$

$$\frac{9x+5}{(x+5)(x-5)(x+5)} \quad \text{or} \quad \frac{9x+5}{(x-5)(x+5)^2}$$

b) $\frac{n-2}{n^2-5n+6} - \frac{n+4}{n^2-11n+30}$

1) Factor

$$\begin{array}{l} \text{product} = 6 \\ \text{add} = -5 \end{array} \left. \vphantom{\begin{array}{l} \text{product} = 6 \\ \text{add} = -5 \end{array}} \right\} -2, -3$$

$$\frac{\cancel{n-2}}{(\cancel{n-2})(n-3)} - \frac{n+4}{(n-5)(n-6)}$$

$$\frac{n^2 - 2n - 3n + 6}{n(n-2) - 3(n-2)}$$

NPVs: $n \neq 2, n \neq 3, n \neq 5, n \neq 6$

$$\frac{(n-2)(n-3)}{\text{product } 30 \left. \vphantom{\text{product } 30} \right\} \begin{array}{l} -5, \\ \text{sum } -11 \end{array} \left. \vphantom{\text{product } 30} \right\} -5, -6$$

$$\frac{1}{n-3} = \frac{n+4}{(n-5)(n-6)}$$

$$\frac{n^2 - 5n - 6n + 30}{n(n-5) - 6(n-5)} = \frac{n^2 - 11n + 30}{(n-5)(n-6)}$$

LCD = $(n-3)(n-5)(n-6)$

$$\frac{1}{n-3} \rightarrow \frac{(n-5)(n-6)}{(n-5)(n-6)} - \frac{(n+4)}{(n-5)(n-6)} \cdot \frac{(n-3)}{(n-3)}$$

$$= \frac{(n-5)(n-6) - (n+4)(n-3)}{(n-3)(n-5)(n-6)}$$

$$= \frac{n^2 - 6n - 5n + 30 - [n^2 - 3n + 4n - 12]}{(n-3)(n-5)(n-6)}$$

$$\frac{n^2 - 11n + 30 - [n^2 + n - 12]}{(n-3)(n-5)(n-6)}$$

$$\frac{\cancel{n^2} - 11n + 30 - \cancel{n^2} - n + 12}{(n-3)(n-5)(n-6)}$$

$$\frac{-12n + 42}{(n-3)(n-5)(n-6)} = \frac{-6(2n-7)}{(n-3)(n-5)(n-6)}$$

$$\frac{2+3}{8+2} = \frac{5}{10}$$

$$= \frac{1}{2}$$

$$\frac{\cancel{2}+3}{\cancel{8}+2} = \frac{3}{8} \neq$$

Example 3 Modelling a Situation with a Rational Expression

A plane travels from Toronto to Vancouver and back a distance of about 3400 km each way. The plane flies 100 km/h faster on the return trip than it does on the journey out. Write, then simplify an expression for the total flying time in terms of the average speed from Toronto to Vancouver.

$$D = (\text{rate})(\text{time})$$

$$D = rt$$

$$D = \frac{rt}{r}$$

$$t = \frac{D}{r}$$

	D	r	t
Tor → Vanc. (going)	3400	r	$\frac{3400}{r}$
Vanc → Tor (return)	3400	r + 100	$\frac{3400}{r+100}$

r = speed on the way to Vancouver

$$\text{Total flying time} = \text{Tor} \rightarrow \text{Vanc. time} + \text{Vanc} \rightarrow \text{Tor time}$$

$$\text{total time} = \frac{3400}{r} + \frac{3400}{r+100}$$

$$\text{Time} = \frac{3400 \cdot \frac{(r+100)}{(r+100)}}{r} + \frac{3400 \cdot \frac{r}{(r+100)}}{(r+100)}$$

$$= \frac{3400r + 340000 + 3400r}{r(r+100)}$$

$$= \frac{6800r + 340,000}{r(r+100)}$$

$$= \frac{6800(r + 50)}{r(r+100)}$$

NPVs:
r ≠ 0
r ≠ -100

$$\text{LCD} = r(r+100)$$

Try: page 565 #5a, 566 #7b, 569 #13b

6.5 Solving Rational Equations

Focus: Solving equations involving rational expressions.

Difference between Expression and Equation?

there IS an "=" sign,
and, you're able to find
a value for what X =
(Solve)

variable and number being multiplied/divided/... together,
but no "=" sign!

(Simplifying)

How to solve Rational Equations?

$$3x(x-8) \left(\frac{1}{x} + \frac{1}{x-8} \right) = \left(\frac{1}{3} \right) 3x(x-8)$$

"Solve for x"

- 1) Factor denominators
- 2) NPVs
- 3) Get LCD
- 4) Multiply each term by LCD, to eliminate All fractions!
- 5) solve.

1) none to do

2) NPVs: $x \neq 0, x \neq 8$

3) LCD: $3x(x-8)$

$$3x(x-8) \left(\frac{1}{x} \right) + 3x(x-8) \left(\frac{1}{x-8} \right) = 3x(x-8) \left(\frac{1}{3} \right)$$

$$3(x-8)(1) + 3x(1) = x(x-8)(1)$$

$$3(x-8) + 3x = x(x-8)$$

$$3x - 24 + 3x = x^2 - 8x$$

$$6x - 24 = x^2 - 8x$$

$$-24 = x^2 - 14x$$

$$0 = x^2 - 14x + 24$$

$$0 = (x-2)(x-12)$$

$$x-2=0$$

$$\boxed{x=2}$$

$$x-12=0$$

$$\boxed{x=12}$$

Solve

Always check to see if your answer is an NPV. If it is, reject it. If all answers get rejected, we say No solution.

(You can check, by substituting into the original equation.)

$$x - 5 = 18$$

$$+5 \quad +5$$

$$x = 23$$

$$\frac{1}{2}(2x) = \frac{1}{2}(10)$$

$$x = 5$$

WT page 578

Example 2 Solving Rational Equations with Binomial Denominators

Solve each equation.

a) $\frac{5}{x+4} = \frac{3}{x-2}$

b) $\frac{x+2}{x-3} = \frac{x-1}{x-2}$

c) $\frac{3x+1}{x^2-1} = \frac{-x}{x+1}$

$$2a) \frac{5}{x+4} = \frac{3}{x-2} \quad \text{NPVs: } x \neq -4$$

$$x \neq 2$$

$$\text{LCD: } (x+4)(x-2)$$

$$5(x-2) = 3(x+4)$$

$$5x - 10 = 3x + 12$$

$$5(x-2) = 3(x+4)$$

$$5x - 10 = 3x + 12$$

$$2x - 10 = 12$$

$$\frac{2x}{2} = \frac{22}{2}, \quad \boxed{x = 11}$$

WT page 579

Example 3 Solving Rational Equations with Binomial and Trinomial Denominators

Solve each equation.

a) $\frac{x}{x-3} = \frac{-6}{x^2-8x+15}$ b) $\frac{x}{x+1} - \frac{x+1}{x-4} = \frac{5}{x^2-3x-4}$

b) $\frac{x}{x+1} - \frac{x+1}{x-4} = \frac{5}{x^2-3x-4}$

~~$(x+1)(x-4)$~~ $\left(\frac{x}{x+1}\right) - \left(\frac{x+1}{x-4}\right) = \left(\frac{5}{(x-4)(x+1)}\right)$

$$x(x-4) - (x+1)(x+1) = 5$$

$$x^2 - 4x - [x^2 + 2x + 1] = 5$$

$$x^2 - 4x - x^2 - 2x - 1 = 5$$

$$-6x - 1 = 5$$

$$-6x = 6$$

$$x = -1$$

~~$x = -1$~~

no solution

Try p 582, $\begin{cases} \# 5b \\ 7b \end{cases}$

1) factor
 $AC = -4$
 $AD + C = -3$
 $-4, +1$
 $x^2 - 4x + (x-4)$
 $x(x-4) + 1(x-4)$
 $(x-4)(x+1)$

2) NPVs: $x \neq 4$
 $x \neq -1$

3) $(x+1)(x-4)$

#5b

$$x^2(2) - \frac{4}{x}x^2 = \frac{6}{x^2}x^2$$

NPV $x \neq 0$
 LCD = x^2

$$2x^2 - 4x = 6$$

not done yet!

$$2x^2 - 4x - 6 = 0$$

Have to factor now:

$$2(x^2 - 2x - 3) = 0$$

$$2(x+1)(x-3) = 0$$

$x = -1$ $x = 3$

#7b) $\frac{x-2}{x+3} = \frac{x+4}{x-1}$ NPV: $x \neq -3$
 $x \neq 1$

LCD: $(x+3)(x-1)$

$$(x+3)(x-1) \left(\frac{x-2}{x+3}\right) = (x+3)(x-1) \left(\frac{x+4}{x-1}\right)$$

$$(x-1)(x-2) = (x+3)(x+4)$$

$$x^2 - 2x - 1x + 2 = x^2 + 4x + 3x + 12$$

$$x^2 - 3x + 2 = x^2 + 7x + 12$$

$$\begin{aligned}
 x^2 - 2x - 1 &= x^2 + 7x + 12 \\
 x^2 - 3x + 2 &= x^2 + 7x + 12 \\
 -x^2 & \\
 -3x + 2 &= 7x + 12 \\
 -7x & \\
 -10x + 2 &= 12 \\
 -2 & \\
 -10x &= 10 \\
 -10 & \\
 x &= -1
 \end{aligned}$$

For next class

- Complete worktext questions for 6.1-6.5
- Complete Trig Booster Exercise worksheet
 - No answers will be posted.
 - Clear, correct and complete work can earn you up to an 8% boost on your Chapter 5 Test mark. (You can't get a mark of over 100% on the test, though!)
 - Due next Thursday, April 13. I will not accept it after that date.
- Reminder - no class Tuesday, April 11