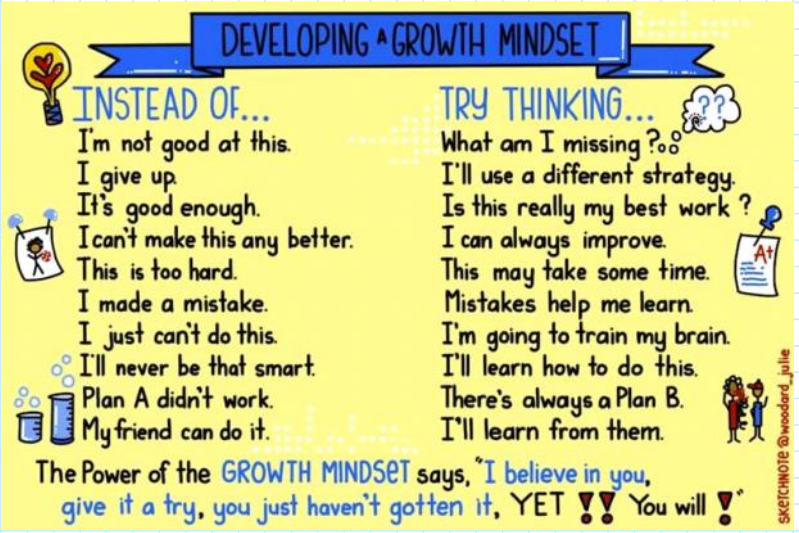
Class_23 Nov 29 - Rational Functions

Sunday, November 20, 2022 3:12 PM

Tonight's Class:

- **Unit 3 Test return/rewrite sign-up**
- **(9.2) Analyzing Rational Functions**
- **(9.3) Rational Equations**



Whiteboards - rational functions so far

Example (TB p 445, #18)

Two stores rent bikes. The first store charges a fixed fee of $20 plus $4/h, and the second store charges a fixed fee of $10 plus $5/h.

a) Write equations for the average cost per hour for each store as a function of the rental time in hours.

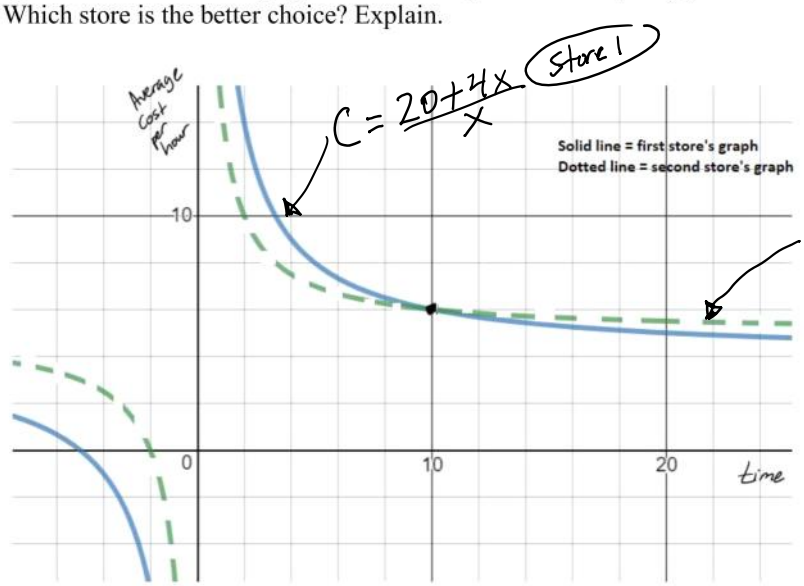
x = # of hours bike is rented

Average cost per hour = $\frac{\text{total cost}}{\text{\# of hours}}$

Store 1 : Average cost = $\frac{\$20 + 4x}{x}$

Store 2 : Average cost = $\frac{\$10 + 5x}{x}$

b) Shown below are the graphs of the two equations from part (a). Which store is the better choice? Explain.



$C = \frac{20+4x}{x}$ (Store 1)

$C = \frac{10+5x}{x}$ (Store 2)

For less than 10 hours, Store 2 costs less

At 10 hours, same cost at either store

More than 10 hours, Store 1 costs less

Practice
(9.1) TB p 442: 2ac, 3cd, 4ac, 5ac, 6, 7bd, 8, 9, 12, 16

Watch the YouTube Videos for these two sections!

Section 9.2 Analyzing Rational Functions https://www.youtube.com/watch?v=rbifC0AmDuM
Section 9.3 Solving Rational Equations https://www.youtube.com/watch?v=xjrG2sE3I5A

9.2 Analyzing Rational Functions

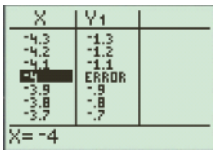
Some rational function equations are more complicated. To analyze and graph them, we ***factor and simplify*** their equations.

Example

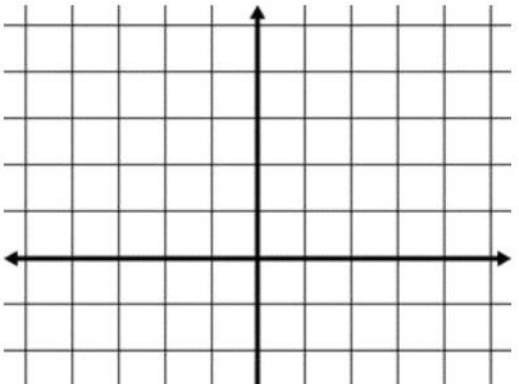
Consider the rational function: $f(x)=\dfrac{x^2+7x+12}{x+4}$

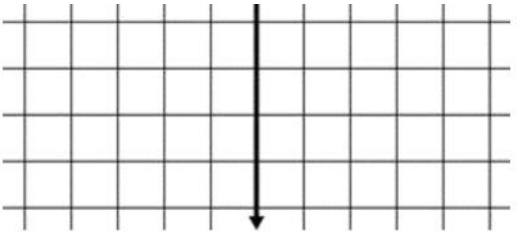
a) Factor and simplify the function's equation.

b) NPV(non-permissible value) =
How does the graph behave near its NPV?



$x \quad f(x)$





Point of Discontinuity (POD) – an ordered pair where the graph of a function does not exist. It occurs whenever the equation's numerator and denominator have a common factor that includes a variable.

Example

a) Complete the table, with the characteristics of the two graphs.

	$f(x)=\frac{x^2+2x-8}{x-2}$	$g(x)=\frac{x^2+6x+8}{x-2}$
Non-permissible value(s)		
Simplified form of equation		
Coordinates of x- and y-intercepts		

b) Graph these rational functions (same as the ones above) using technology. Below each equation draw a rough sketch of its graph.

$$f(x)=\frac{x^2+2x-8}{x-2} \qquad g(x)=\frac{x^2+6x+8}{x-2}$$

When does a rational function have

- ***a point of discontinuity***
- ***a vertical asymptote?***

Whiteboards - PODs and horizontal asymptotes

- **Horizontal asymptotes questions #1 and #2, we can get the h.a. equations from remembering the two base graphs we learned**
- **Horizontal asymptotes questions #3-6, these equations are not in the form of the base graphs, but we can get the h.a. equations from looking at the graphs**
- **Horizontal asymptotes questions #7 - remember how to get the h.a. equation when the equations are more unusual?**

Key Ideas for Rational Function Graphs

1) Horizontal Asymptotes

Find the degree of the numerator and denominator.

Numerator degree < Denominator degree ***horizontal asymptote equation:*** $y = 0$
Numerator degree = Denominator degree ***horizontal asymptote equation:*** $y = \frac{leading\ coefficient\ of\ num}{leading\ coefficient\ of\ denom}$
Numerator degree > Denominator degree Graph will have a ***slant asymptote***

2) NPVs, PODs, and vertical asymptotes

Factor numerator and denominator completely.

- Set ***each factor of the denominator = 0***, to get all NPVs.
  - Is there a factor that cancels with a factor in the numerator? It gives the x-value of a POD.
  - Is there a factor that doesn't cancel with a numerator factor? It gives the location of a vertical asymptote.

3) Intercepts

- ***y*-intercepts –** substitute $x = 0$ into the function (either the original or the simplified form) and solve for y
- ***x*-intercepts** - set each factor of the simplified numerator = 0 and solve for x

4) Sketch

- Plot all x-intercepts and y-intercepts
- Show points of discontinuity (PODs) as "holes", using an open circle
- Show all asymptotes as dotted lines.
- Find more points on the graph, as needed, by substituting into its equation.
- Make sure graph does not cross any vertical asymptotes.

To try:

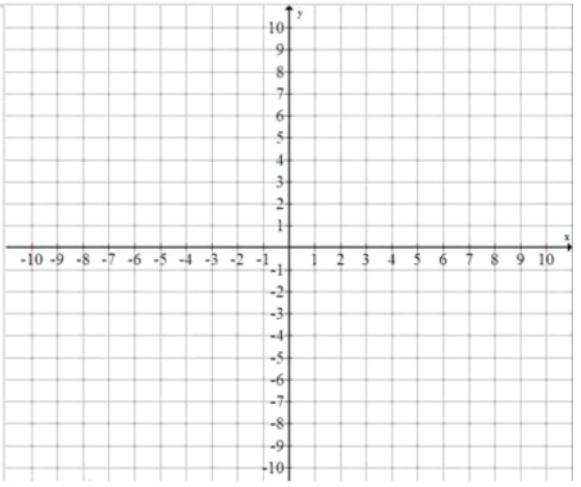
Original Equation	Factored form of equation	List all NPVs, and for each one identify if it gives a POD or a vertical asymptote. - Find the (*x, y*) coordinates of each POD . - Find the equation of each vertical asymptote.	Horizontal asymptote equation or say "Slant"
$y = \frac{2x+10}{x^2+2x-15}$			
$y = \frac{2x^2+7x+6}{x^2-2x-8}$			
$y = \frac{x^2+3x-4}{x-1}$			

Characteristics of Rational Functions - group worksheet

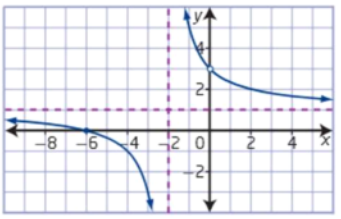
Without using technology, accurately sketch the function's graph: $y = \dfrac{x^2 + 3x - 10}{x^2 - 4}$

Give the values of the graph's:

- NPVs
- asymptote equations
- coordinates of PODs
- *x*- and *y*-intercepts



Example (TB page 453, #7a)
Write the equation of the pictured rational function.



9.3 Connecting Graphs and Rational Equations

To solve rational equations algebraically:

- Determine the value of all non-permissible values. List them.
- Find the least-common denominator (LCD).
- Multiply each term in the equation by the LCD, to eliminate fractions
- Solve this simpler equation. If a solution is an NPV, reject it.

Example

a) Solve algebraically: $\frac{3}{x} = 1 + \frac{x-13}{6}$

b) Verify the solution graphically.

SKIPPING THIS

There are two ways to solve graphically.

1) Graph Y_1 = LHS of equation

Graph Y_2 = RHS of equation.

Find the x-values where the 2 graphs intersect.

OR

2) Collect all terms of the original equation on one side of the equals sign.

Graph this equation.

Find all of this graph's x-intercepts (zeroes)

To try:

1a) Find the roots of this rational equation, algebraically: $x+\frac{6}{x+2}-5=0$

b) Verify, graphically. **SKIPPING THIS**

2a) Find the roots of this rational equation, algebraically: $1+\frac{2}{x}=\frac{x}{x+3}$

b) Verify the solution graphically.

For next class, Thursday, December 1

- **Complete the Chapter 9 Hand-in**
- **Do more questions from tonight's in-class group worksheets, in the areas where you know you need more practice. (Each worksheet is posted, along with full solutions)**

Practice

(9.1) TB p 442: 2ac, 3cd, 4ac, 5ac, 6, 7bd, 8, 9, 12, 16
(9.2) TB p 452: 4-7, 8ac, 11, 14
(9.3) TB p 465: 1, 2, 3-6(ac), 9, 11

Coming up

- **Tuesday, Dec 6 - Chapter 9 Test**
- **Thursday, Dec 8 - Unit 4 Test**
- **Tuesday, Dec 13 - optional class, for Unit 4 rewrites**