Class_25 June 15 - Sequences and Series, Infinite Series

Wednesday, June 14, 2023 4:28 PM

Tonight's Class:

- **Chapter 9 Hand-in due today**
- **Sequences and Series - Should we go through anything here? (G.1-G.2)**
- **Infinite Series (G.3)**

Notes package, page 17:

5. Between the Canadian censuses in 2001 and 2006, the number of people who could speak in Cree had increased by 7%. In 2006, 87 285 people could converse in Cree. Assume the 5-year increase continues to be 7%. To the nearest hundred, how many people will be able to converse in Cree in 2036?

$r = 1 + 0.07$
$= 1.07$

87 285 ← in 2006 (t_1)

87 285(1.07) ← in 2011 (t_2)

. . . . ?

2016 (t_3), 2021 (t_4), 2026 (t_5), 2031 (t_6), 2036 (t_7)

$$\boxed{t_n = ar^{n-1}}$$

$t_7 = 87285(1.07)^6 = (130991.2488)$

$\doteq 131{,}000$ people

$$\boxed{\begin{array}{r} 2036 \\ -2006 \\ \hline 30 \end{array} \quad 30 \div 5 = 6 \qquad t_1 + 6 \text{ more terms} \Rightarrow t_7}$$

Two geometric series

$4 + 12 + 36 + 108 + 324 + \cdots$

$a = 4$
$r = 3$

$S_1 = 4$

$S_2 = 4 + 12 = 16$

$S_3 = 4 + 12 + 36 = 52$

$S_4 = 4 + 12 + 36 + 108 = 160$

$S_5 = 4 + 12 + 36 + 108 + 324 = 484$

$S_{10} = ?$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{10} = \frac{4(1-3^{10})}{1-3} = 118096$$

$$S_{100} = \frac{4(1-3^{100})}{1-3} = 1.03 \times 10^{48}$$

<u>A different series</u>:

$$4 + 1 + \frac{1}{4} + \frac{1}{16} + \cdots$$

$a = 4$
$r = \frac{1}{4}$

$S_1 = 4$

$S_2 = 5$

$S_3 = 5\frac{1}{4}$ or 5.25

$S_4 = 5.3125$

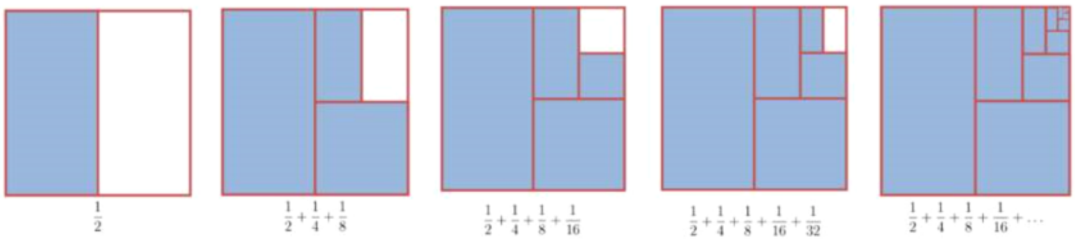
$S_5 = 5.328125$

$\vdots$

$$S_{10} = \frac{4(1-(\frac{1}{4})^{10})}{1-\frac{1}{4}} = 5.333328247$$

$$S_{100} = \frac{4\left(1-\left(1/4\right)^{100}\right)}{1-1/4} \doteq 5.\overline{3}$$

G. 3 <u>Infinite Geometric Series</u>



Consider the series illustrated above: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$

- What do you notice about the terms? are getting smaller
- What do you think the sum is? → 1

An ***infinite geometric series*** is one with an infinite number of terms – there is no last term.

With an infinite series, we will find that either:

Successive terms get smaller and smaller and smaller.

- The common ratio, r, has a value between -1 and 1. $-1 < r < 1$
- This type of series is called ***convergent.***
- We **CAN** find the sum by using the formula: $\boxed{S=\frac{a}{1-r}}$

Successive terms stay the same size, or continue to get larger and larger

- The common ratio, r, satisfies either $r \geq 1$ *or* $r \leq -1$.
- This type of series is called ***divergent***.
- We **CANNOT** find the sum of this series. We say "No finite sum."

A geometric series has a **finite sum**, or **converges**, only when $-1 < r < 1$

$$S=\frac{a}{1-r}\text{, }\mathbf{only\ true\ if\ -1 < r < 1}$$

If you have an infinite geometric series where $r \geq 1$ or $r \leq -1$, it will <u>not</u> have a finite sum, it will **diverge.**

$\boxed{S=\frac{a}{1-r},\ -1<r<1}$

1) find r
2) if $-1<r<1$, do the sum using $S=\frac{a}{1-r}$

To Try:

1. Determine whether each infinite geometric series has a finite sum. If it does have a finite sum, find its value.

a) $10-\frac{20}{3}+\frac{40}{9}-\ldots$

$r=\frac{-20/3}{10}=-\frac{20}{3}\cdot\frac{1}{10}=\frac{-20}{30}=-\frac{2}{3}$ ✓ can use formula

$S=\frac{10}{1-(-2/3)}=\frac{10}{1+2/3}=\frac{10}{3/3+2/3}=\frac{10}{5/3}=\frac{10}{1}\cdot\frac{3}{5}=\frac{30}{5}=\boxed{6}$

b) $0.1+0.3+0.9+\ldots$

$r=\frac{0.3}{0.1}$

$r=3$ ✗ no, can't use the formula!

$\boxed{\text{No finite sum}}$

c) $32+8+2+0.5+\ldots$

$r=\frac{8}{32}$

$r=1/4$ ✓

$S=\frac{32}{1-1/4}=\frac{32}{3/4}=\frac{32}{1}\cdot\frac{4}{3}=\boxed{\frac{128}{3}\text{ or }42\tfrac{2}{3}}$

d) $-1-\frac{3}{4}-\frac{9}{16}-\frac{27}{64}\ldots$

$r=\frac{-3/4}{-1}$

$r=3/4$ ✓

$S=\frac{-1}{1-3/4}=\frac{-1}{1/4}=-1\cdot\frac{4}{1}=\boxed{-4}$

2a) The sum of an infinite geometric series is 63 and the first term is 21. Find the common ratio.

sum $=S$, first term $=a$

$S=\frac{a}{1-r}$

$(1-r)\times(63)=\left(\frac{21}{\cancel{1-r}}\right)\times\cancel{(1-r)}$

$63-63r=21$

$-63r=21-63$

$\frac{-63r}{-63}=\frac{-42}{-63}$

$r=\frac{42}{63}\ \frac{\div 7}{\div 7}$

$r=\frac{6}{9}=\boxed{\frac{2}{3}}$

b) The sum of an infinite geometric series is 24/7 and $r=-\frac{3}{4}$. Find the first term, a.

$S=\frac{a}{1-r}$

$\frac{24}{7}=\frac{a}{1-(-3/4)}$

$\frac{24}{7}=\frac{a}{1+3/4}$

$7\cdot\left(\frac{24}{7}\right)=\left(\frac{a}{7/4}\right)7$

$\frac{7}{4}(24)=\left(\frac{7a}{7/4}\right)\cdot\frac{7}{4}$

$\frac{42}{7}=\frac{7a}{7}$

$\boxed{a=6}$

c) An infinite geometric series has the terms $a=32$ and $t_4=4$. Determine the sum of the infinite series.

$S=\frac{a}{1-r}$

$t_n=ar^{n-1}$

$t_4=ar^3=4$

$\frac{32r^3}{32}=\frac{4}{32}$

$r^3=1/8$

$\sqrt[3]{r^3}=\sqrt[3]{1/8}$

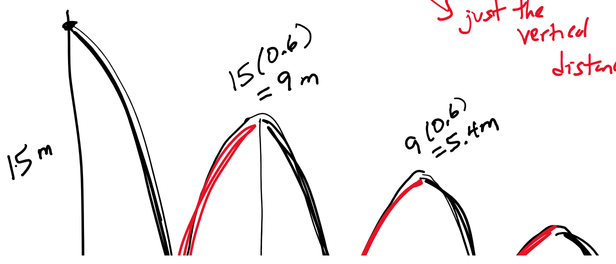
$r=\frac{1}{2}$

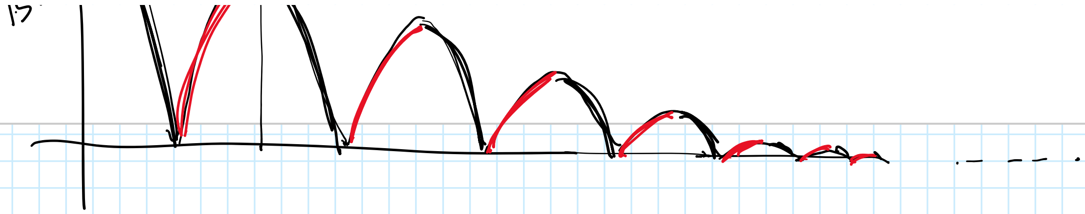
$S=\frac{32}{1-1/2}=\frac{32}{1/2}=32\cdot\frac{2}{1}=\boxed{64}$

4. A ball is dropped from a height of 15 meters and bounces to 60% of the previous height. How far has the ball traveled when it ***comes to rest***?

means (infinite series)

just the vertical distance: up + down.





$$\text{Total} = (\text{initial drop}) + \left(\begin{array}{c}\text{all the up}\\ \text{distances}\end{array}\right) + \left(\begin{array}{c}\text{all the matching}\\ \text{down distance}\end{array}\right)$$

$$= 15 + (9 + 5.4 + \dots) + (9 + 5.4 + \dots)$$

$$r = \frac{5.4}{9}$$

$$r = 0.6$$

$$S = \frac{9}{1-r} = \frac{9}{1-0.6} = \frac{9}{0.4} = 22.5\text{ m}$$

$$= 15 + 22.5 + 22.5 = 15 + 2(22.5)$$

$$= \boxed{60\text{ m}}$$

<u>Optional worksheet:</u>
More Sequences & Series Practice

<u>Coming up</u>

- **Tuesday, June 20**
  - **Test 7 (9.3, G.1-G.4)**
  - **Chapter G (10) Hand-in due**
- **Wednesday, June 21**
  - **Rewrite day (optional, can do up to 2 test rewrites)**