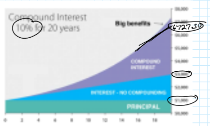
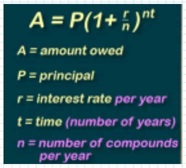
Class_26 Apr 25 - Annuities: Investments & Loans

Tuesday, April 11, 2023 2:33 PM

Tonight's Class:

- **Any questions from last class? (7.1)**
- **Working through sections 7.2-7.4**
  - **Compound Interest**
  - **Comparing Simple and Compound Interest**
  - **Annuities: Investments and Loans**
- **Work on Chapter 7 hand-in and practice questions from worktext**
- **Next class is our last class - Chapter 7 Test**





$1000

simple interest
at 10% interest
for 20 years

$A = P(1+rt)$

$A = \$1000(1+(0.10)(20))$

$\boxed{A = \$3000}$

r = rate, as decimal

t = time in years

Compound interest

$A = P(1+\frac{r}{n})^{nt}$

$A = 1000(1+\frac{0.10}{1})^{1\times 20}$

$= 1000(1+0.10)^{20}$ compounded annually

$= \boxed{\$6727.50}$

n = # of times it's compounded each year

t = time in years (20 years)

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P t $r = 0.072$ $A = P(1+\frac{r}{n})^{nt}$

3. A principal of $5000 is invested for 5 years at 7.2% interest. Determine the amount for each compounding period.

a) annually ← n=1

$A = 5000(1+\frac{0.072}{1})^{(1\times 5)}$

$= 5000(1.072)^5$

$\boxed{\$7078.54}$

b) monthly ← how many months in one year? $n=12$

$A = 5000(1+\frac{0.072}{12})^{(12\times 5)}$

$= \boxed{\$7158.94}$

c) bi-weekly every 2 weeks

n=26

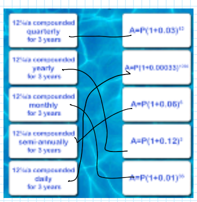
$A = 5000(1+\frac{0.072}{26})^{(26\times 5)}$

$= \boxed{\$7163.08}$

d) daily ← $n = 365$

$A = 5000(1+\frac{0.072}{365})^{(365\times 5)}$

$= \boxed{\$7166.39}$



$A = P(1+\frac{r}{n})^{nt}$

12% ⇒ r=0.12
quarterly ⇒ n=4
3 years ⇒ t=3
$A = P(1+\frac{0.12}{4})^{(4\times 3)}$
$A = P(1+0.03)^{12}$

12%
semi-annually ⇒ n=2
3 years
$A = P(1+\frac{0.12}{2})^{2\times 3}$

12%
yearly ⇒ n=1
3 years
$A = P(1+\frac{0.12}{1})^{1\times 3}$

12%
daily ⇒ n=365
3 years
$A = P(1+\frac{0.12}{365})^{365\times 3}$

12%
monthly ⇒ n=12
3 years
$A = P(1+\frac{0.12}{12})^{12\times 3}$

	n
annually	1
semi-annually	2
quarterly	4
monthly	12
bi-monthly	24
bi-weekly	26
weekly	52
daily	365

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Example 2 Calculating the Amount and the Principal

a) A principal of $4500 is invested in a savings account that pays 2.25% interest compounded quarterly. What is the amount in the account after 3.5 years?

b) Suppose you need to save $5000 to buy a car on your 21st birthday, in 5 years' time. You find an online investment that pays 4.15% interest compounded semi-annually. What principal do you need to invest now?

$A = P(1+\frac{r}{n})^{nt}$

a) $P = 4500$
$r = 2.25\% = \boxed{0.0225}$
$n = 4$
$t = 3.5$

$A = 4500(1+\frac{0.0225}{4})^{(4\times 3.5)}$

$= \boxed{\$4867.63}$

b) $P = ?$
$A = 5000$
$r = 4.15\% = \boxed{0.0415}$
$n = 2$
$t = 5$

$\frac{A}{(1+\frac{r}{n})^{nt}} = \frac{P\cancel{(1+\frac{r}{n})^{nt}}}{\cancel{(1+\frac{r}{n})^{nt}}}$

$P = \frac{A}{(1+\frac{r}{n})^{nt}}$

$A = 5000$

$r = 4.15\% = \boxed{0.0415}$

$n = 2$

$t = 5$

$\left(1+\frac{r}{n}\right)^{nt}$ $\quad$ $\cancel{\left(1+\frac{r}{n}\right)^{nt}}$

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

$$= \frac{5000}{\left(1+\frac{0.0415}{2}\right)^{(2\times 5)}}$$

$$= \boxed{\$4071.70}$$

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Example 3 Calculating the Interest and the Interest Rate

a) A principal of $1000 is invested for 4 years at an interest rate of 10.2% compounded weekly. Calculate the interest earned.

b) A principal of $5000 is invested for 5 years and earns interest of $1235. The interest rate is compounded monthly. Calculate the annual interest rate, to 2 decimal places.

a) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$P = 1000$

$t = 4$

$r = 10.2\% = 0.102$

$n = 52$

$$A = 1000\left(1+\frac{0.102}{52}\right)^{(52\times 4)}$$

$$= \$1503.21$$

But, the interest is: $\boxed{\text{final amount} - \text{principal}}$

$1503.21 - 1000$

$= \$503.21$

b) $P = \$5000$

$t = 5$

interest $= \$1235$

$n = 12$

$r = ?$

$A = P + I$

$= 5000 + 1235$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\frac{6235}{5000} = \frac{\cancel{5000}}{\cancel{5000}}\left(1+\frac{r}{12}\right)^{(12\times 5)}$$

$$\sqrt[60]{\frac{6235}{5000}} = \sqrt[60]{\left(1+\frac{r}{12}\right)^{60}}$$

$$\sqrt[60]{\frac{6235}{5000}} \underset{-1}{} = 1 \underset{-1}{} + \frac{r}{12}$$

$$12\left(\sqrt[60]{\frac{6235}{5000}} - 1\right) = \left(\frac{r}{12}\right)\cdot 12$$

$$12\left(\sqrt[60]{\frac{6235}{5000}} - 1\right) = r$$

OR $\quad r = 12\left(\left(\frac{6235}{5000}\right)^{1/60} - 1\right)$

$r = 0.04422\ldots$

$\boxed{4.4\%}$

When you invest money, you may want to know how long it will take your investment to double. The rule of 72, below, provides an estimate of the time it will take to double your money when the interest is compounded.

Rule of 72

Time in years for an investment to double

$$= \frac{72}{\text{Annual interest rate as a percent}}$$

(not decimal)

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Example 4 Using the Rule of 72

a) Determine the number of years it will take to double an investment when it earns interest at 6% compounded annually.

b) Suppose you leave high school at 18 and have saved $5000. You want to have $20 000 by the age of 30. What average annual interest rate do you need to achieve your goal? Is this rate likely? Explain.

a) $t = \frac{72}{\text{annual interest rate}} = \frac{72}{6} = 12$ years

b) \$5000, if it doubled we'd have \$10,000 — 6 years

\$10,000 " " " " \$20,000 — 6 years

$30 - 18 = 12$ years

$$6 \text{ years} = \frac{72}{\text{rate}}$$

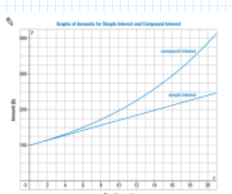
$r \cdot 6 = \frac{72}{r} \cdot r$

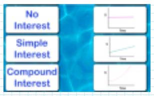
$\frac{6r}{6} = \frac{72}{6}$, $\boxed{r = 12\%}$

not likely ☹

7.3 Comparing Simple Interest and Compound Interest

Focus: Investigate graphs of simple and compound interest and compare their trends.





Pink – compound bi-weekly (26)
Yellow – compound monthly (12)
Blue – compound semi-annually (4)
Red – simple

7.4 Annuities: Investments and Loans

Focus: Investigate how annuities are applied to investments and loans.

Annuity – what is it?

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An annuity is a series of equal deposits made at equal time intervals. Each deposit is made at the end of each time interval.

Annuity

From Wikipedia, the free encyclopedia

For other uses, see Annuity (disambiguation).

An **annuity** is a series of payments made at equal intervals.[1] Examples of annuities are regular deposits to a savings account, monthly home mortgage payments, monthly insurance payments and pension payments. Annuities can be classified by the frequency of payment dates. The payments (deposits) may be made weekly, monthly, quarterly, yearly, or at any other regular interval of time. Annuities may be calculated by mathematical functions known as "annuity functions".

Ordinary Simple Annuity

An **ordinary simple annuity** has the following characteristics:

- Payments are made at the end of the payment intervals, and the payment and compounding frequencies are equal.
- The first payment occurs one interval after the beginning of the annuity.
- The last payment occurs on the same date as the end of the annuity.

For example, most car loans are ordinary simple annuities where payments are made monthly and interest rates are compounded monthly. As well, car loans do not require the first monthly payment until the end of the first month.

< https://math.libretexts.org/Bookshelves/Applied_Mathematics/Business_Math_(Olivier)/11%3A_Compound_Interest_Annuities/11.01%3A__Fundamentals_of_Annuities >

What two numbers must be the same for the annuity formulas to be valid? What has to be done if these numbers are not the same?

For the formulas to be valid, the payment period, that is, how often a payment is made; and the compounding period for the interest rate must be the same.
If these periods are not the same, an effective interest rate has to be calculated and used in the formulas.

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Amount of an Annuity FUTURE VALUE of the annuity

The amount of an annuity is:

$$A = \frac{R[(1+i)^n - 1]}{i}$$

- A is the amount in dollars.
- R is the regular deposit or payment in dollars.
- i is the interest rate per compounding period, as a decimal.
- n is the number of deposits or payments.

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Example 1 Calculating the Amount of an Annuity

A regular deposit of $500 is invested in a savings account at the end of each month.
The interest rate is 3.15% compounded monthly. → 12 times a year
What is the amount of the annuity at the end of 25 years? Future value.

$$A = \frac{R[(1+i)^n - 1]}{i}$$

n = # of deposits/payments, in total

i = interest rate per compounding period

$A = ?$
$R = \$500$
$i = \frac{0.0315}{12}$
$n = 12 \times 25 = 300$

$$A = \frac{500\left[\left(1 + \frac{0.0315}{12}\right)^{300} - 1\right]}{\left(\frac{0.0315}{12}\right)}$$

$$\boxed{A = \$227,738.71}$$

(notice, I paid \$500 × 300 = \$150,000)

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Example 2 Calculating the Regular Deposit for an Annuity

Suppose you want to retire at age 65 with $1 000 000. At age 30, you start investing a monthly deposit in a stock portfolio that pays 3.6% interest compounded monthly. What should your monthly deposit be to achieve your goal?

future

65 − 30

= 35 years

$A = 1{,}000{,}000$

$R = ?$

$i = \frac{0.036}{12}$

n = total # of payments

$35 \times 12 = 420$

$$(i)(A) = \left(\frac{R\left[(1+i)^n - 1\right]}{\cancel{i}}\right)\cancel{(i)}$$

$$R\left[\quad\right] = (A)(i)$$

$$\boxed{R = \frac{(A)(i)}{\left[(1+i)^n - 1\right]}}$$

$$R = \frac{(1{,}000{,}000)\left(\frac{0.036}{12}\right)}{\left[\left(1+\frac{0.036}{12}\right)^{420} - 1\right]} = \boxed{\$1191.05}$$

The present value of an annuity refers to **how much money would be needed today to fund a series of future annuity payments**. Because of the time value of money, a sum of money received today is worth more than the same sum at a future date.

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Present Value of an Annuity

The present value of an annuity is: $PV = \frac{R[1-(1+i)^{-n}]}{i}$

- PV is the present value in dollars.
- R is the regular payment in dollars.
- i is the interest rate per compounding period, as a decimal.
- n is the number of payments.

Notes: The payment period is the same as the compounding period. The payments are made at the end of the compounding periods.

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Example 3 Calculating the Present Value and Regular Payments of an Annuity

a) A person buys a computer. She pays $75 a month for 48 months at an interest rate of 12% compounded monthly. What is the present value of the loan?

$PV = ?$

$R = \$75$

$i = \frac{0.12}{12} = 0.01$

$n = 48$

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$= \frac{75\left[1-(1+0.01)^{-48}\right]}{0.01}$$

$$= \boxed{\$2848.05}$$

present value

How much does the person actually pay, total?

$\$75 \times 48$

$= \boxed{\$3600}$

PV

b) A person buys a car for $35 750. The loan is repaid over 5 years. The dealership offers an interest rate of 0.5% annually. What is the annual repayment on the loan?

$PV = 35{,}750$

$n = 5$

$R = ?$

$i = 0.005$

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$R = \frac{(PV)(i)}{\left[1-(1+i)^{-n}\right]}$$

$$R = \frac{(35{,}750)(0.005)}{\left[1-(1+0.005)^{-5}\right]} = \boxed{\$7257.61}$$

(person actually paid

7257.61
× 5

$36,288.03

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Example 4 Solving Problems Involving Mortgages and Annuities

a) A mortgage on a home is $500 000. The effective interest rate on the loan is 3.6% compounded monthly, for 25 years. What is the approximate monthly payment for this mortgage?

used when the payment period is different from the compounding period.

$PV = 500{,}000$

$i = \frac{0.036}{12} = 0.003$

$n = 25 \times 12 = 300$

$$R = \frac{(PV)(i)}{\left[1-(1+i)^{-n}\right]}$$

$$R = \frac{(500\,000)(0.003)}{\left[1-(1+0.003)^{-300}\right]}$$

$$R = \boxed{\$2530.01}$$

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b) A person has saved $200 000 in investments for retirement. The investment earns 4.0% interest, compounded quarterly. At the age of 65, how much money can the person withdraw quarterly to have enough money until the age of 90?

$R = ?$

$PV = \$200\,000$

$i = \frac{0.04}{4} = 0.01$

$n = 25 \times 4 = 100$

$90 - 65 = 25$ years

$$(i)(PV) = \left(\frac{R[1-(1+i)^{-n}]}{\cancel{i}}\right)(i)$$

$$(i)(PV) = R[1-(1+i)^{-n}]$$

$$\frac{(i)(PV)}{[1-(1+i)^{-n}]} = R$$

$$R = \frac{(200\,000)(0.01)}{[1-(1+0.01)^{-100}]} = \boxed{\$3173.15}$$

<u>**For next class**</u>

- **Complete the Chapter 7 Hand-in to help you prepare for the Chapter 7 Test. You will have the finance formula sheet to use while writing the test.**
- **Next class will be our last class!**