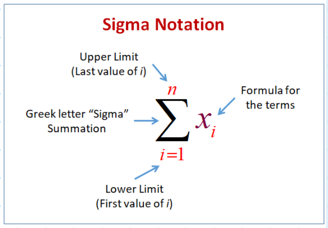
Class_26 June 19 - Sigma Notation

Plan For Today:

1. Question about anything from last class (G.1-3)?
2. Finish Topic 10: Geometric Sequences & Series
   - G.1 Geometric Sequences
   - G.2 Geometric Series
   - G.3 Infinite Geometric Series
   - **G.4 Sigma Notation**
3. Work on practice questions in handouts.
4. Review for Test 7 (Ch9 and G10)
5. Any review of older tests for rewrites on Wednesday?



Plan Going Forward:

1. Practice working through G.4 questions and review.
2. Last test tomorrow (maybe last class) - REWRITE DAY WEDNESDAY

- **TOPIC 10 (G) ASSIGNMENT DUE ON TUESDAY, JUNE 20TH**
- **TEST 7 ON 9.3-10.4 ON TUESDAY, JUNE 20TH**

Please let us know if you have any questions or concerns about your progress in this course. The notes from today will be posted at egolfmath.weebly.com after class.
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Review?

#4 G assign

$t_3 = 10 \qquad t_6 = 80 \qquad t_{10} =$

$\times r \quad \times r \quad \times r$

$r^3 = \frac{80}{10}$

$\sqrt{r^3} = \sqrt{8}$

$r = 2$

$\sqrt{r^2}$ $\quad \sqrt[3]{r^3} = r$

$\sqrt[4]{r^4}$ $\quad \sqrt[5]{r^5} = r$

$\sqrt[6]{r^6}$ $\quad \sqrt[7]{r^7} = r$

$\pm$

$\sqrt[3]{-8} = -2$

$a?$

t_3

$ar^{3-1} = 10$

$\frac{a(2)^2}{4} = \frac{10}{4}$

$a = \frac{10}{4} \rightarrow a = \frac{5}{2}$

$t_{10} = ar^{n-1}$

$= \left(\frac{10}{4}\right)(2)^{10-1}$

$=$

Extra practice #4

4b) $t_3 = 5 \qquad t_6 = 135$

$\times r \quad \times r \quad \times r$

$r^3 = \frac{135}{5}$

$\sqrt[3]{r^3} = \sqrt[3]{27}$

$r = 3$

$a \rightarrow$

$t_6 = \qquad t_3 =$

$a(3)^5 = 135 \qquad a(3)^2 = 5$

$\frac{135}{}$ $\qquad 9a = 5$

$a(3) = 135$

$r = 3$

$\frac{135}{243}$ =

same 'a'

$9a = 5$

$a = \frac{5}{9}$

$t_n = ar^{n-1}$

$t_n = \frac{5}{9}(3)^{n-1}$

ex: $t_{10} = \frac{5}{9}(3)^{10-1}$

$= \frac{5}{9}(3)^9$

$= \frac{5}{9}(19683)$

$= 5(2187)$

$\boxed{t_{10} = 10,935}$

4 a) $a = 4$ $\quad \underbrace{t_{13}} = 16384$

$ar^{13-1} = 16384$

$\frac{4r^{12}}{4} = \frac{16384}{4}$

$r^{12} = 4096$

$\sqrt[12]{r^{12}} = \sqrt[12]{4096}$

$r = 2$

common base

$r^{12} = 2^{12}$

$r = 2$

$t_n = ar^{n-1}$

$t_n = 4(2)^{n-1}$

#6

#6

6. A population of rabbits is growing at a rate of 8% a year. If there are 160 rabbits in the initial population, create a general term equation, t_n, describing this sequence. Use it to find the number of rabbits after 6 years.

$r = 1 + 0.08$ not 0.08

$\underline{r = 1.08}$

$\underline{a = 160}$

$t_n = 160(1.08)^{n-1}$

$t_6 = 160(1.08)^{6-1}$

$t_6 = 235.0924928$

$t_6 = 235$ rabbits.

#10

$$S = 0.3 + \frac{2}{100} + \frac{2}{1000} + \frac{2}{10000} + \dots$$

$\cancel{a = 0.3}$ add into sum

$r = \frac{1}{15}$, $r = \frac{1}{10}$, $r = \frac{1}{10}$

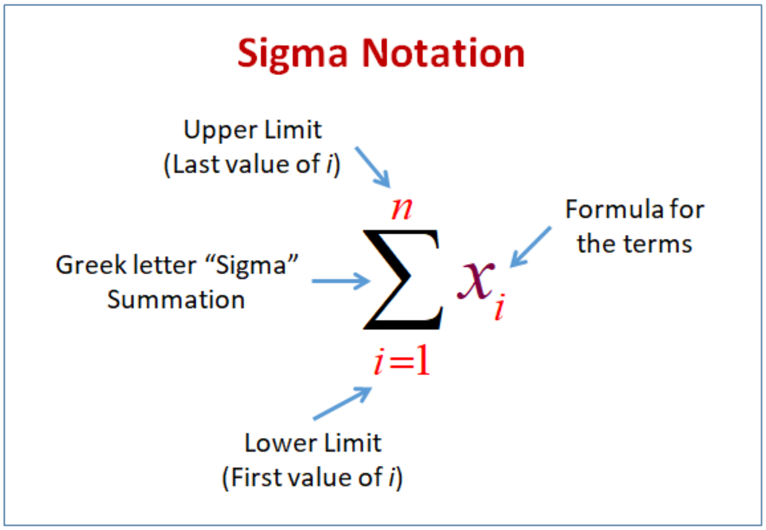
$a = \frac{2}{100}$

geometric

$$S_\infty = \frac{a}{1-r} + \frac{3}{10}$$

$$= \frac{\frac{2}{100}}{1 - \frac{1}{10}} + \frac{3}{10}$$

G.4 Sigma Notation



Sigma Notation

There is a special notation that is used to represent a series. For example, the geometric series $3 + 6 + 12 + 24 + 48 + 96$ has 6 terms, with first term 3 and common ratio 2. The general term is $t_n = 3(2)^{n-1}$.

Each term in the series can be expressed in this form.

$t_1 = 3(2)^{1-1}$ $\quad t_2 = 3(2)^{2-1}$ $\quad t_3 = 3(2)^{3-1}$

$t_4 = 3(2)^{4-1}$ $\quad t_5 = 3(2)^{5-1}$ $\quad t_6 = 3(2)^{6-1}$

The series is the sum of all these terms, and is represented as shown.

The sum of … $\longrightarrow \displaystyle\sum_{k=1}^{6} 3(2)^{k-1} \longleftarrow$ … all numbers of the form $3(2)^{k-1}$ …

… for integral values of k from 1 to 6.

The symbol Σ is the capital Greek letter sigma, which corresponds to S, the first letter of the word "sum." When Σ is used as shown above, it is called *sigma notation*. In sigma notation, k is frequently used as the variable under the Σ sign and in the expression following it. Any letter can be used, as long as it is not used elsewhere.

Example 1:

Finite geometric sequence: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{32768}$

Related finite geometric series: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{32768}$

Written in sigma notation: $\sum_{k=1}^{15} \frac{1}{2^k}$

Example 2:

Infinite geometric sequence: $2, 6, 18, 54, \ldots$

Related infinite geometric series: $2 + 6 + 18 + 54 + \ldots$

Written in sigma notation: $\sum_{n=1}^{\infty} (2 \cdot 3^{n-1})$

$$\sum_{k=0}^{10-1} \tfrac{1}{2}\left(\tfrac{1}{2}\right)^k = \frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^{10}}{1-\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(\frac{1-\frac{1}{1024}}{\frac{1}{2}}\right)$$

$$= 1 - \frac{1}{1024}$$

$$= 0.9990234375$$

1) Find the sum of the infinite series:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

As first term equals a_1, $a_1 = \frac{1}{2}$

Using $a_n = r a_{n-1}$ and first two terms, $r = \frac{1}{2}$

$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{k-1} = \frac{a_1}{1-r} = \frac{(1/2)}{1-(1/2)} = 1$$

2) Find the sum of $\sum_{k=1}^{\infty} 4\left(-\frac{1}{3}\right)^{k-1}$

$$a_1 = 4$$

$$r = -\frac{1}{3}$$

$$\therefore$$

$$\sum_{k=1}^{\infty} 4\left(-\frac{1}{3}\right)^{k-1} = \frac{a_1}{1-r} = \frac{4}{1-(-1/3)} = \frac{4}{(4/3)} = 4 \cdot \frac{3}{4} = 3$$

G. 4 Sigma Notation

Series are sums. We have been writing them in an expanded form, like this example:

$32+8+2+0.5$

Here is a different way to write the same series: $\sum_{n=3}^{6} 32(0.25)^{n-3}$

This notation, ***sigma notation,*** uses the upper-case Greek letter sigma, Σ , and means we need to "SUM UP" all the terms in the series generated by the expression after the sigma. Sigma notation is used in many mathematics and science resources, so it's useful to understand how it works.

To ***EXPAND*** a series written in sigma notation

- Substitute the bottom number – the one written below the sigma – into the expression. This gives the first term of the series.
- Keep substituting in consecutive numbers to get the next terms of the series.
- The last number to substitute in is the one written above the sigma. This gives the last term of the series.

$$\sum_{n=3}^{6} 32(0.25)^{n-3} = 32(0.25)^{3-3}+32(0.25)^{4-3}+32(0.25)^{5-3}+32(0.25)^{6-3}$$

To Try:

end at n=4

sub n into here + evaluate to get each term

Σ means SUM ∴ + b/w terms

1. Expand and evaluate: $\sum_{n=2}^{4} 3\left(\frac{1}{2}\right)^{n-1} = 3\left(\frac{1}{2}\right)^{2-1} + 3\left(\frac{1}{2}\right)^{3-1} + 3\left(\frac{1}{2}\right)^{4-1}$

start at n=2

$$3\left(\tfrac{1}{2}\right) + 3\left(\tfrac{1}{2}\right)^2 + 3\left(\tfrac{1}{2}\right)^3$$

$$\tfrac{3}{2} + 3\left(\tfrac{1}{4}\right) + 3\left(\tfrac{1}{8}\right)$$

$$\underbrace{\tfrac{3}{2} + \tfrac{3}{4} + \tfrac{3}{8}}_{\text{series}} = \frac{12+6+3}{8} = \boxed{\tfrac{21}{8}}$$

LCD

SUM

2. Evaluate: $\sum_{k=5}^{21} 4(2)^{k-2}$

only need first 3 terms

$$= 4(2)^{5-2} + 4(2)^{6-2} + 4(2)^{7-2} + \ldots \underbrace{4(2)^{21-2}}_{\text{extra}}$$

$$= 32 \underbrace{+ 64}_{r=2} \underbrace{+ 128}_{r=2} + \ldots$$

✓ geometric

$n = 21-5+1$

$n=17$

$a=32$

sum formula.

$$S_n = \frac{a(1-r^n)}{1-r} \rightarrow = \frac{32(1-2^{17})}{1-2} \rightarrow = \boxed{4{,}194{,}272}$$

13. Determine the sum, if possible:

$S_n = \frac{a(1-r^n)}{1-r}$

a) $\sum_{i=1}^{\infty} -4\left(\frac{4}{5}\right)^{i}$

b) $\sum_{i=1}^{6} 2(3)^{i}$

+ #7 on G - Assign.

a) $\sum_{i=1} -4\left(\frac{4}{5}\right)$ b) $\sum_{i=1} 2(3)$

c) $\sum_{i=1}^{\infty} 5\left(\frac{4}{3}\right)^i$ $\leftarrow S_n = \frac{a(1-r^n)}{1-r}$ d) $\sum_{i=1}^{\infty} 5\left(\frac{2}{3}\right)^i$

G - Assign.

$$\sum_{i=1}^{\infty} -4\left(\frac{4}{5}\right)^i = -4\left(\frac{4}{5}\right)^1 - 4\left(\frac{4}{5}\right)^2 - 4\left(\frac{4}{5}\right)^3 - \ldots$$

$$= -\frac{16}{5} - 4\left(\frac{16}{25}\right) - 4\left(\frac{64}{125}\right)$$

$$= \underbrace{-\frac{16}{5} - \frac{64}{25}}_{r=\frac{4}{5}} \underbrace{- \frac{256}{125}}_{r=\frac{4}{5}} \ldots$$

$$S_\infty = \frac{a}{1-r} = \frac{-\frac{16}{5}}{1-\frac{4}{5}} \leftarrow \frac{5}{5} - \frac{4}{5}$$

$$= \frac{-\frac{16}{5}}{\frac{1}{5}}$$

$$= \frac{-16}{\cancel{5}} \times \frac{\cancel{5}}{1}$$

$$\boxed{S_\infty = -16}$$

HW #7, 13, 14 →

To ***WRITE*** a geometric series in sigma notation

- Figure out the values of a and r
- Write a general expression for the series, using the format: $t_n = ar^{n-1}$
- Determine how many terms you have, n. Use this for the number above the sigma.

# of terms in series

term equation

$$\sum_{i=1}^{n} ar^{i-1}$$

Example

Write the given series in sigma notation: $5+15+45+135+405+1215+3645$

1. $a=$ $\quad a=5$
2. $r=$ $\quad r=\frac{15}{5} \Rightarrow r=3$
3. $n=$ $\quad n=7$
4. Sigma notation $\quad \sum_{i=1}^{7} ar^{i-1} \rightarrow \boxed{\sum_{i=1}^{7} 5(3)^{i-1}}$

To Try

1a) Write the series: $3+9+27+81+243$ in sigma notation.

$a=3$ $\quad r=3$ $\quad n=5$

$$\boxed{\sum_{i=1}^{5} 3(3)^{i-1}}$$

b) Write the series: $20+40+80+\ldots+163\,840$ in sigma notation.

$a=20$ $\quad r=2$ $\quad n=?\rightarrow n=14 \rightarrow \boxed{\sum_{i=1}^{14} 20(2)^{i-1}}$

$t_n = ar^{n-1}$

$163840 = 20(2)^{n-1}$

÷ both sides by 20

common base

log method.

$2^{13} = 2^{n-1} \leftarrow 8192 = 2^{n-1} \rightarrow \log 8192 = (n-1)\log 2$

$13 = n-1$ $\quad$ +1

$n=14$

$\frac{\log 8192}{\log 2} = \frac{(n-1)\log 2}{\log 2}$

$\frac{\log 8192}{\log 2} = n-1$ $\quad$ +1

$n = \frac{\log 8192}{\log 2} + 1$

$n=14$

2. Determine each sum. $\rightarrow$ $a=$

2. Determine each sum. $\rightarrow$ $a=$ $r=$ $n=$

a) $\sum_{k=4}^{12} 8\left(\frac{1}{2}\right)^{k-2}$

$a = 8\left(\frac{1}{2}\right)^{4-2}$

$a = 8\left(\frac{1}{2}\right)^{2}$

$a = 8\left(\frac{1}{4}\right)$

$a = 2$

$r = \frac{1}{2}$ ✓

$t_2 = 8\left(\frac{1}{2}\right)^{5-2}$

$= 8\left(\frac{1}{2}\right)^{3}$

$= 8\left(\frac{1}{8}\right)$

$t_2 = 1 \rightarrow r = \frac{1}{2}$

$n = 12-4+1$

$n = 9$

$S_9 = \frac{2\left(1-\left(\frac{1}{2}\right)^9\right)}{1-\frac{1}{2}}$

$= \frac{2\left(1-\frac{1}{512}\right)}{\frac{1}{2}}$ $\qquad \frac{512}{512}-\frac{1}{512} = \frac{511}{512}$

$2 \times \frac{2}{1}$

$= \cancel{4}^{1}\left(\frac{511}{\cancel{512}_{128}}\right)$

$\boxed{S_9 = \frac{511}{128}}$

b) $\sum_{n=7}^{17} (2)^n$

2. Find the sum of these series, correct to two decimal places.

a) $\sum_{k=1}^{\infty} 100(0.3)^{k-1}$

determine 'r' to see if it's a infinite sum

$\rightarrow a = 100(0.3)^{1-1}$

$= 100(0.3)^0$

$= 100(1)$

$a = 100$

$r = 0.3$

check.

$t_2 = 100(0.3)^{2-1}$

$= 100(0.3)$

$= 30$

$r = \frac{30}{100} \rightarrow 0.3$ * use S_∞

$n = \infty$

$S_\infty = \frac{100}{1-0.3}$

$= \frac{100}{0.7} \rightarrow \boxed{142.86}$

$S_\infty = \frac{100}{\frac{7}{10}}$

$= 100 \times \frac{10}{7}$

$= \boxed{\frac{1000}{7}}$

b) $\sum_{k=1}^{\infty} \frac{4}{5}\left(\frac{2}{3}\right)^{n-1}$

$a = \frac{4}{5}\left(\frac{2}{3}\right)^{1-1}$

$a = \frac{4}{5}$

$t_2 = \frac{4}{5}\left(\frac{2}{3}\right)^{2-1}$

$t_2 = \frac{4}{5}\left(\frac{2}{3}\right)$

$= \frac{8}{15}$

$r = \frac{\frac{8}{15}}{\frac{4}{5}} \rightarrow \frac{\cancel{8}^{2}}{\cancel{15}_{3}} \times \frac{\cancel{5}^{1}}{\cancel{4}_{1}} \rightarrow \frac{2}{3}$

infinite sum formula

$S_\infty = \frac{\frac{4}{5}}{1-\frac{2}{3}} \rightarrow \boxed{2.40}$

$= \frac{\frac{4}{5}}{\frac{3}{3}-\frac{2}{3}}$

$= \frac{\frac{4}{5}}{\frac{1}{3}}$

$= \frac{4}{5} \times \frac{3}{1}$

$= \boxed{\frac{12}{5}} \rightarrow \boxed{2.40}$

C_26 More Sequences and Series Practice

Tuesday, December 3, 2019 6:17 PM

C_25 More Sequences and Series Practice

(Solutions at right)

Sequences and Series – more practice

1. Is the following sequence geometric?

a) 10, 15, 22.5, 37.5,...

b) 7, 14, 21, 28,...

2. Find the common ratio, r, of each geometric sequence

a) $-1, -5, -25, -125, \ldots$

b) $-200, 100, -50, -25, \ldots$

3. Find the next three terms of the following sequence

a) 386561, 55223, 7889, ____, ____, ____

b) $-\frac{1}{5}, -\frac{1}{15}, -\frac{1}{45}$, ____, ____, ____

4. Find a formula for the nth term of each geometric sequence.

a) $a = 4$, $t_{13} = 16384$

b) $t_3 = 5$, $t_6 = 135$

5. The seventh term of a geometric sequence is 1215 and the fourth term is 45. Find the common ratio, then find the value of the ninth term.

6. A population of rabbits is growing at a rate of 8% a year. If there are 160 rabbits in the initial population, create a general term equation, t_n, describing this sequence. Use it to find the number of rabbits after 6 years.

7. Find the sum of the following geometric series. If necessary, round to 2 decimal places.

a) $729 - 243 + 81 - 27 + \ldots$ (10 terms)

b) $7 + 14 + 28 + 56 + \ldots + 7168$

c) $\sum_{n=4}^{10} 5(2)^n$

8. Find the common ratio of a geometric series with a first term of 38 and a sum to infinity of 76.

Sequences and Series – more practice

1. Is the following sequence geometric?

a) 10, 15, 22.5, 37.5,... $r = 1.5$

$\frac{15}{10} = 1.5 \quad \frac{22.5}{15} = 1.5$ } yes

b) 7, 14, 21, 28,... no. To get the next term, we're adding the same number, not multiplying by the same number.

2. Find the common ratio, r, of each geometric sequence

a) $-1, -5, -25, -125, \ldots$ $\boxed{r = 5}$

$r = \frac{-5}{-1} = 5$

b) $-200, 100, -50, -25, \ldots$ $r = \frac{100}{-200} = -\frac{1}{2}$

$\boxed{r = -\frac{1}{2}}$

3. Find the next three terms of the following sequence

a) 386561, 55223, 7889, 1127, 161, 23

$r = \frac{55223}{386561} = \frac{1}{7}$

b) $-\frac{1}{5}, -\frac{1}{15}, -\frac{1}{45}, \frac{-1}{135}, \frac{-1}{405}, \frac{-1}{1215}$

$r = \frac{-1/15}{-1/5} = -\frac{1}{15} \cdot \frac{5}{-1} = \frac{-5}{-15} = \frac{1}{3}$

4. Find a formula for the nth term of each geometric sequence.

a) $a = 4$, $t_{13} = 16384$

$t_{13} = ar^{12}$

$16384 = 4(r^{12})$

$4096 = r^{12}$

$2^{12} = r^{12} \quad r = 2$

$\boxed{t_n = 4(2)^{n-1}}$

b) $t_3 = 5$, $t_6 = 135$

$\frac{135}{5} = \frac{ar^5}{ar^2} \quad r^3 = 27 \quad r = 3$

$ar^2 = 5$

$a = \frac{5}{9}$

$\boxed{t_n = \frac{5}{9}(3)^{n-1}}$

5. The seventh term of a geometric sequence is 1215 and the fourth term is 45. Find the common ratio, then find the value of the ninth term.

$t_7 = 1215 = ar^6$

$t_4 = 45 = ar^3$

$27 = r^3$, $r = 3$

$ar^3 = 45$

$a = \frac{45}{r^3}$

$a = \frac{45}{27} = \frac{5}{3}$

$t_9 = \frac{5}{3}(3)^8$

$\boxed{t_9 = 10\,935}$

6. A population of rabbits is growing at a rate of 8% a year. If there are 160 rabbits in the initial population, create a general term equation, t_n, describing this sequence. Use it to find the number of rabbits after 6 years.

160, 172.8, ---

1 yr, 2 yrs

$t_n = 160(1.08)^{n-1}$

$t_7 = 160(1.08)^6$

$\doteq \boxed{253 \text{ rabbits}}$

7. Find the sum of the following geometric series. If necessary, round to 2 decimal places.

a) $729 - 243 + 81 - 27 + \ldots$ (10 terms) $r = \frac{-243}{729} = -\frac{1}{3}$

$S_{10} = \frac{729(1 - (-\frac{1}{3})^{10})}{1 - (-\frac{1}{3})} \doteq \boxed{546.74}$

b) $7 + 14 + 28 + 56 + \ldots + 7168$

$7168 = 7(2)^{n-1}$

$1024 = 2^{n-1}$

$2^{10} = 2^{n-1} \Rightarrow n = 11$

$S_{11} = \frac{7(1 - 2^{11})}{1-2} = 14329$

c) $\sum_{n=4}^{10} 5(2)^n$

\# of terms: $10 - 4 + 1 = 7$ terms

$= 5(2)^1 + 5(2)^2 + \ldots$

$= 80 + 160 + \ldots$ $r = \frac{160}{80} = 2$

$S_7 = \frac{80(1 - 2^7)}{1-2} = \boxed{10160}$

8. Find the common ratio of a geometric series with a first term of 38 and a sum to infinity of 76.

$S = \frac{a}{1-r}$

$(1-r)(76) = \left(\frac{38}{\cancel{1-r}}\right)\cancel{(1-r)}$

$76(1-r) = 38$

$76 - 76r = 38$

$38 = 76r$

$\frac{38}{76} = r$, $\boxed{r = \frac{1}{2}}$

9. Find the general term, t_n, for the described sequences:

a) geometric, beginning: $-2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots$

b) geometric, with $t_3 = 75$ and $r = 5$

c) geometric, with $t_4 = 5$ and $r = \frac{1}{4}$

10. Find the 25th term of the following geometric sequence: $2, 2\sqrt{3}, 6, \ldots$

11. List the first five terms of the geometric sequence with $t_3 = 8$ and $r = -\frac{1}{2}$.

12. Find the requested sum for each geometric sequence.

a) Find S_{12} correct to 2 decimal places, for $a = 5$, $r = \frac{2}{3}$

b) Find S_9 for $a = -3$ and $r = 2$

c) Find the sum of the first 11 terms of the geometric series that begins $7 - 14 + 28 - \ldots$

13. Determine the sum, if possible:

a) $\sum_{i=1}^{\infty} -4\left(\frac{4}{5}\right)^i$

b) $\sum_{i=1}^{6} 2(3)^i$

c) $\sum_{i=1}^{\infty} 5\left(\frac{4}{3}\right)^i$

d) $\sum_{i=1}^{\infty} 5\left(\frac{2}{3}\right)^i$

14. A helium balloon rises 80 meters the first minute after it is released. Each minute after that it rises 15% less than the previous minute. How high does the balloon rise in total?

9. Find the general term, t_n, for the described sequences:

a) geometric, beginning: $-2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots$

$\boxed{t_n = -2(-\tfrac{1}{2})^{n-1}}$

b) geometric, with $t_3 = 75$ and $r = 5$

$t_3 = 75 = a(5)^2$

$75 = 25a, \quad a = 3$

$\boxed{t_n = 3(5)^{n-1}}$

c) geometric, with $t_4 = 5$ and $r = \frac{1}{4}$

$t_4 = 5 = a(\tfrac{1}{4})^3 \qquad 5 = a(\tfrac{1}{64})$

$a = 320$

$\boxed{t_n = 320(\tfrac{1}{4})^{n-1}}$

10. Find the 25th term of the following geometric sequence: $2, 2\sqrt{3}, 6, \ldots$

$r = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$t_{25} = 2(\sqrt{3})^{24} = 2(3^{12}) = 1062882$

11. List the first five terms of the geometric sequence with $t_3 = 8$ and $r = -\frac{1}{2}$.

$8 = a(-\tfrac{1}{2})^2 \qquad 8 = a(\tfrac{1}{4}) \qquad a = 32$

$\boxed{32, -16, 8, -4, 2}$

12. Find the requested sum for each geometric sequence.

a) Find S_{12} correct to 2 decimal places, for $a = 5$, $r = \frac{2}{3}$

$S_{12} = \frac{5(1 - (\frac{2}{3})^{12})}{1 - \frac{2}{3}} = 14.88$

b) Find S_9 for $a = -3$ and $r = 2$

$S_9 = \frac{-3(1 - (2)^9)}{1 - 2} = -1533$

c) Find the sum of the first 11 terms of the geometric series that begins $7 - 14 + 28 - \ldots$

$S_{11} = \frac{7(1 - (-2)^{11})}{1 - (-2)} = 4781$

13. Determine the sum, if possible:

a) $\sum_{i=1}^{\infty} -4\left(\frac{4}{5}\right)^i$

$= -4(\tfrac{4}{5}) + -4(\tfrac{4}{5})^2 + \ldots$

$= -\frac{16}{5} - \frac{64}{25} \ldots \quad r = \frac{-64/25}{-16/5}$

$r = 4/5$

a) $S = \frac{-16/5}{(1 - 4/5)} = \frac{-16/5}{4/5} = -16$

b) $\sum_{i=1}^{6} 2(3)^i$

$= 2(3) + 2(3)^2 + \ldots$

$= 6 + 18 + \ldots$

$S_6 = \frac{6(1 - 3^6)}{1 - 3} = 2184$

c) $\sum_{i=1}^{\infty} 5\left(\frac{4}{3}\right)^i$

$= 5(\tfrac{4}{3}) + 5(\tfrac{4}{3})^2 + \ldots$

$= \frac{20}{3} + \frac{80}{9} + \ldots \quad r = \frac{80/9}{20/3} = \frac{4}{3}$

no sum, $r > 1$

d) $\sum_{i=1}^{\infty} 5\left(\frac{2}{3}\right)^i$

$= 5(\tfrac{2}{3}) + 5(\tfrac{2}{3})^2$

$= 10/3 + \frac{20}{9} + \ldots$

$r = \frac{20/9}{10/3} = 2/3 \longrightarrow S = \frac{10/3}{1 - 2/3} = \frac{10/3}{1/3} = 10$

14. A helium balloon rises 80 meters the first minute after it is released. Each minute after that it rises 15% less than the previous minute. How high does the balloon rise in total?

$80, 68, 57.8 \ldots$

$S = \frac{80}{1 - 0.85} \doteq 533.33 \text{ m}$

Test 7 Review

C_26 Key and T7 Practice Test Rationals and Series

Test 7 Review

C_25 Unit 4 Practice Test Rationals and Series

(Solutions at right)

Unit 4 Practice Test

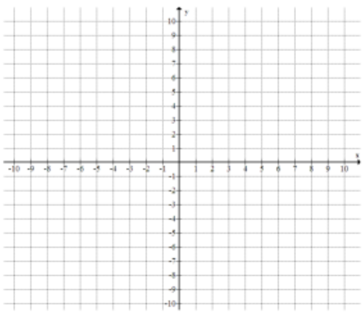
omit for Test 7

1. Given the original rational function $y = \frac{1}{x}$ and the transformed function, $y = \frac{5}{x+4} - 2$:

a) List the transformations taking place.

b) Complete the tables below. For the first table, give 6 points found on the graph of the original function $y = \frac{1}{x}$. In the second and third tables, show the transformed points after stretches/reflections, and finally after translations. Write the mapping notation in the headings of each table.

x	y



c) Accurately sketch the final transformed function. Include its asymptotes, using dotted lines. Label each asymptote with its equation.

d) Find the coordinates of the final graph's x-intercept and y-intercept.

Unit 4 Practice Test

1. Given the original rational function $y = \frac{1}{x}$ and the transformed function, $y = \frac{5}{x+4} - 2$:

a) List the transformations taking place.

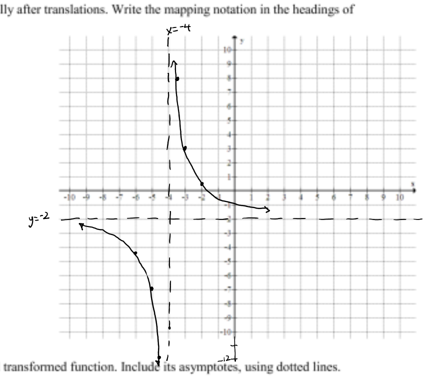
VS 5
4 left
2 down

b) Complete the tables below. For the first table, give 6 points found on the graph of the original function $y = \frac{1}{x}$. In the second and third tables, show the transformed points after stretches/reflections, and finally after translations. Write the mapping notation in the headings of each table.

x	y
-2	-1/2
-1	-1
-1/2	-2
1/2	2
1	1
2	1/2

x	5y
-2	-5/2 = -2.5
-1	-5
-1/2	-10
1/2	10
1	5
2	5/2 = 2.5

x-4	5y-2
-6	-4.5
-5	-7
-4 1/2	-12
-3 1/2	8
-3	3
-2	0.5



c) Accurately sketch the final transformed function. Include its asymptotes, using dotted lines. Label each asymptote with its equation.

d) Find the coordinates of the final graph's x-intercept and y-intercept.

x-int, let y = 0

$0 = \frac{5}{x+4} - 2$

$2 = \frac{5}{x+4}$

$2(x+4) = 5$

$2x + 8 = 5$

$2x = -3$

$x = -\frac{3}{2}$

$(-\frac{3}{2}, 0)$

y-int, let x = 0

$y = \frac{5}{0+4} - 2$

$y = \frac{5}{4} - 2\left(\frac{4}{4}\right)$

$y = \frac{5}{4} - \frac{8}{4}$

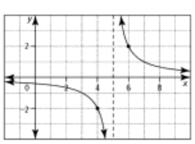
$y = -\frac{3}{4}$

$(0, -\frac{3}{4})$

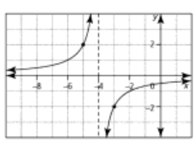
OMIT

2. Write the equation of each function in the form $y = \frac{a}{x-h} + k$

a)



b)



OMIT

3. Given the original function $y = \frac{1}{x^2}$ and the transformed function $y = \frac{1}{x^2 + 6x + 9}$. What is the transformation that takes place?

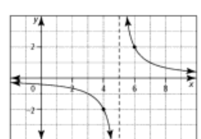
4. Does the graph of the rational function $y = \frac{(x+a)(x+b)}{(x+b)}$ have a vertical asymptote or a point of discontinuity?

5. Complete the table:

	$y = \frac{(x+3)(x-2)}{(x+5)(x+3)}$
Non-permissible value(s)	
Equation of vertical asymptote	
Coordinates of POD	
Equation of horizontal asymptote	

2. Write the equation of each function in the form $y = \frac{a}{x-h} + k$

a)

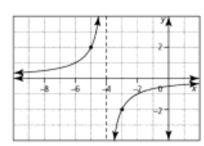


$y = \frac{a}{x-5}$, contains (6,2)

$2 = \frac{a}{6-5}$ $2 = \frac{a}{1}$, a=2

$\boxed{y = \frac{2}{x-5}}$

b)



$y = \frac{a}{x+4}$, contains (-3,-2)

$-2 = \frac{a}{-3+4}$, $-2 = \frac{a}{1}$, a=-2

$\boxed{y = \frac{-2}{x+4}}$

3. Given the original function $y = \frac{1}{x^2}$ and the transformed function $y = \frac{1}{x^2 + 6x + 9}$. What is the transformation that takes place?

$y = \frac{1}{x^2+6x+9} = \frac{1}{(x+3)(x+3)}$

$= \frac{1}{(x+3)^2}$

Comparing $y = \frac{1}{x^2}$ and $y = \frac{1}{(x+3)^2}$, we've moved $\boxed{\text{3 left}}$

4. Does the graph of the rational function $y = \frac{(x+a)\cancel{(x+b)}}{\cancel{(x+b)}}$ have a vertical asymptote or a point of discontinuity?

point of discontinuity

5. Complete the table:

	$y = \frac{\cancel{(x+3)}(x-2)}{(x+5)\cancel{(x+3)}}$ OR $y = \frac{x^2+x-6}{x^2+8x+15}$
Non-permissible value(s)	$x \neq -5$, $x \neq -3$
Equation of vertical asymptote	$x = -5$
Coordinates of POD	$(-3, -\frac{5}{2})$
Equation of horizontal asymptote	degree top = 2 degree bottom = 2 $\Rightarrow y = \frac{1}{1}$, $\boxed{y=1}$

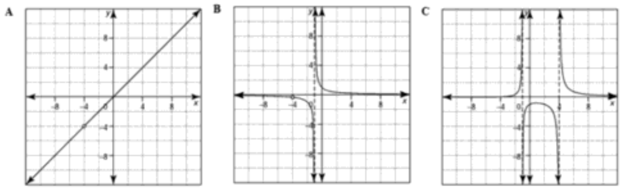
Use x = -3 in simple equation:

$y = \frac{x-2}{x+5} = \frac{-3-2}{-3+5}$

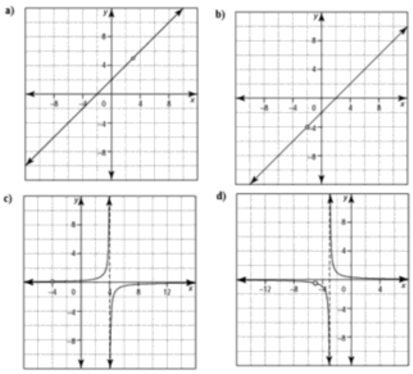
$= \frac{-5}{2}$

6. Without using technology, match the equation of each rational function with the appropriate graph.

a) $y = \frac{x+4}{x^2-3x-4}$ b) $y = \frac{x+4}{x^2+5x+4}$ c) $y = \frac{x^2+4x}{x+4}$



7. Write the equation for each graphed rational function.



6. Without using technology, match the equation of each rational function with the appropriate graph.

a) $y = \frac{x+4}{x^2-3x-4} = \frac{x+4}{(x+1)(x-4)}$

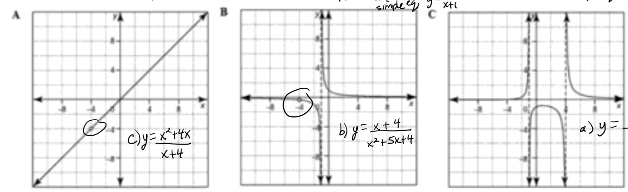
v.a. $x=-1$, $x=4$

b) $y = \frac{x+4}{x^2+5x+4} = \frac{\cancel{x+4}\ 1}{(\cancel{x+4})(x+1)}$

POD at $x=-4$, simple eq. $y = \frac{1}{x+1}$

c) $y = \frac{x^2+4x}{x+4} = \frac{x(\cancel{x+4})}{\cancel{x+4}} \Rightarrow$ POD at $x=-4$

simple equation, $y = x$



7. Write the equation for each graphed rational function.

a) POD $x=3$, so we need $x-3$ in top + bottom

x-int. $x=-2$, so we need $x+2$ in top

$\boxed{y = \frac{(x-3)(x+2)}{x-3}}$

b) POD $x=-2$

$\boxed{y = \frac{(x+2)(x-2)}{(x+2)}}$

x intercept at $x=2$, so we need $x-2$ in top

c) v.a. $x=4$, so we need $x-4$ in bottom

POD $x=-4$, so $x+4$ in top + bottom

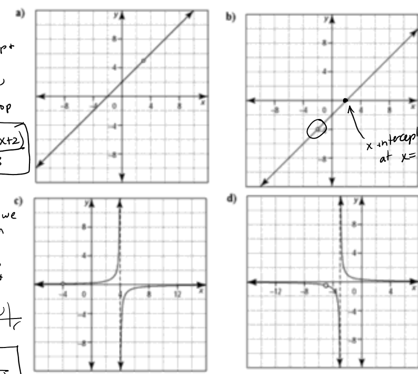
it's reflected: so, need v.s of -1.

$\boxed{y = \frac{-1(x+4)}{(x+4)(x-4)}}$

d) v.a. $x=-3$, so we need $x+3$ in denominator

POD $x=-5$, so $x+5$ in top + bottom

$\boxed{y = \frac{x+5}{(x+5)(x+3)}}$



8. Write the equation of a possible rational function that has a vertical asymptote at $x = 2$, a point of discontinuity at $x = -2.5$, and passes through the point $(6,-3)$

9. Solve algebraically. List all restrictions/NPVs.

a) $\frac{18}{x^2-9} + 1 = \frac{x}{x+3}$

NPV $x \neq \pm 3$

$(x+3)(x-3)\left[\frac{18}{(\cancel{x+3})(\cancel{x-3})} + 1 = \frac{x}{\cancel{x+3}}\right]$

$18 + (x+3)(x-3) = x(x-3)$

$18 + \cancel{x^2} - 9 = \cancel{x^2} - 3x \rightarrow \frac{9}{-3} = \frac{-3x}{-3}$

reject

$x \neq -3$

final answer = $\boxed{\text{no solution}}$

b) $\frac{3}{5x-2} + x = 5$

10. Solve graphically.

a) $3x = \frac{6x}{2x-5}$

b) $\frac{17-3x+x^2}{x-1} = 2x-5$

8. Write the equation of a possible rational function that has a vertical asymptote at $x = 2$, a point of discontinuity at $x = -2.5$, and passes through the point $(6,-3)$

$y = \frac{a(x+2.5)}{(x-2)(x+2.5)}$

Substitute $(6,-3)$:

$-3 = \frac{a(6+2.5)}{(6-2)(6+2.5)}$

$-3 = \frac{a(\cancel{8.5})}{(4)(\cancel{8.5})}$

$-3 = \frac{a}{4}$, $a = -12$

$y = \frac{-12(x+2.5)}{(x-2)(x+2.5)}$

9. Solve algebraically. List all restrictions/NPVs.

a) $\frac{18}{x^2-9} + 1 = \frac{x}{x+3}$

NPVs: $x \neq -3$, $x \neq 3$

$(x+3)(x-3)\left[\frac{18}{(x+3)(x-3)} + 1 = \frac{x}{x+3}\right]$

$\cancel{(x+3)}\cancel{(x-3)}\left(\frac{18}{\cancel{(x+3)}\cancel{(x-3)}}\right) + (x+3)(x-3)(1) = (x\cancel{+3})(x-3)\left(\frac{x}{\cancel{x+3}}\right)$

$18 + x^2 \cancel{-3x} \cancel{+3x} - 9 = x(x-3)$

$\cancel{x^2} + 9 = \cancel{x^2} - 3x$

$\frac{9}{-3} = \frac{-\cancel{3}x}{\cancel{-3}}$

$x = -3$, but that's an NPV

$\Rightarrow \boxed{\text{no solution}}$

b) $\frac{3}{5x-2} + x = 5$

NPV $5x-2 \neq 0$, $5x \neq 2$, $x \neq \frac{2}{5}$

$(5x-2)\left[\frac{3}{5x-2} + x = 5\right]$

$\cancel{(5x-2)}\left(\frac{3}{\cancel{5x-2}}\right) + (5x-2)(x) = (5x-2)(5)$

$3 + 5x^2 - 2x = 25x - 10$

$5x^2 - 2x + 3 - 25x + 10 = 0$

$5x^2 - 27x + 13 = 0$

Use quadratic formula to solve

$x = \frac{27 \pm \sqrt{(-27)^2 - (4)(5)(13)}}{2(5)}$

$x = \frac{27 \pm \sqrt{729 - 260}}{10}$

$x = \frac{27 \pm \sqrt{469}}{10}$

$\doteq \boxed{4.87}$

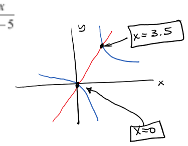
$\doteq \boxed{0.53}$

10. Solve graphically.

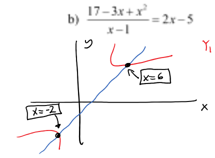
a) $3x = \frac{6x}{2x-5}$

$Y_1 = 3x$

$Y_2 = \frac{6x}{2x-5}$



b) $\frac{17-3x+x^2}{x-1} = 2x-5$



$Y_1 = \frac{17-3x+x^2}{x-1}$

$Y_2 = 2x-5$

11. Is each sequence geometric? If it is, what is the common ratio, r?

a) 1, 2, 4, 8, 16… b) 0.6, 0.06, 0.006 c) 2, 4, 6, 10, 16…

12. State the common ratio, then list the next 3 terms of each geometric sequence.

a) 1, 3, 9, 27, … b) 5, -15, 45, -135, … c) $6, 2, \frac{2}{3}, \frac{2}{9}, \ldots$

13. For each geometric sequence, write a formula for t_n and use it to find the indicated term.

a) 0.01, 0.08, 0.64, 5.12, . . . Give t_n formula. Use it to find t_6.

b) 12 000, 8400, 5880, 4116, . . . Give t_n formula. Use it to find t_8.

14. Use the S_n formula to find the **sum** of the first 10 terms of each geometric series.

a) 20 + 60 + 180 + 540 + . . .

b) 50 + 37.5 + 28.125 + 21.09375 + . . .

c) 2.5 – 6.25 + 15.625 – 39.0625 + . . .

15. A doctor prescribes 200 mg of medication on the first day of treatment. The dosage is halved on each successive day. The medication lasts for seven days. To the nearest milligram, what is the total amount of medication administered?

11. Is each sequence geometric? If it is, what is the common ratio, r?

a) 1, 2, 4, 8, 16… b) 0.6, 0.06, 0.006 c) 2, 4, 6, 10, 16…

yes, $r = 2$ yes, $r = 0.1$ not geometric

12. State the common ratio, then list the next 3 terms of each geometric sequence.

a) 1, 3, 9, 27, … b) 5, -15, 45, -135, … c) $6, 2, \frac{2}{3}, \frac{2}{9}, \ldots$

$r=3$ 81, 243, 729

$r=-3$ 405, -1215, 3645

$r=\frac{1}{3}$ $\frac{2}{27}, \frac{2}{81}, \frac{2}{243}$

13. For each geometric sequence, write a formula for t_n and use it to find the indicated term.

a) 0.01, 0.08, 0.64, 5.12, . . . Give t_n formula. Use it to find t_6.

$r = \frac{0.08}{0.01} = 8$

$t_n = 0.01(8)^{n-1}$

$t_6 = 0.01(8)^5 = 327.68$

b) 12 000, 8400, 5880, 4116, . . . Give t_n formula. Use it to find t_8.

$r = \frac{8400}{12000} = 0.7$

$t_n = 12000(0.7)^{n-1}$

$t_8 = 12000(0.7)^7 = 988.2516$

14. Use the S_n formula to find the **sum** of the first 10 terms of each geometric series.

$S_n = \frac{a(1-r^n)}{1-r}$

a) 20 + 60 + 180 + 540 + . . .

$S_{10} = \frac{20(1-3^{10})}{1-3} = 590\,480$

b) 50 + 37.5 + 28.125 + 21.09375 + . . .

$S_{10} = \frac{50(1-0.75^{10})}{-0.75} \doteq 188.74$

c) 2.5 – 6.25 + 15.625 – 39.0625 + . . .

$S_{10} = \frac{2.5(1-(-2.5)^{10})}{1-(-2.5)} \doteq -6811.25$

15. A doctor prescribes 200 mg of medication on the first day of treatment. The dosage is halved on each successive day. The medication lasts for seven days. To the nearest milligram, what is the total amount of medication administered?

$S_7 = \frac{200(1-0.5^7)}{1-0.5} \doteq 397$ mg

16. A student band has two choices of payment to receive for 50 hours of performance.
Choice 1: \$0.10 for the first hour, \$0.12 for the second hour, \$0.144 for the third hour, etc.
Choice 2: \$100 for the first 10 hours, \$200 for the second 10 hours, \$400 for the third 10 hours, etc.
Which set of payments should the students choose to receive?

17. If the infinite geometric series has a finite sum, find the sum.

a) 81 + 27 + 9 + 3 + . . .

b) 140 + 35 + 8.75 + 2.1875 + . . .

c) 20 + 30 + 45 + 67.5 + . . .

d) 50 – 25 + 12.5 – 6.25 + . . .

18. Write each series using sigma notation.

a) 40 + 20 + 10 + 5 + 2.5 b) 3 + 9 + 27 + 81 + 243 + 729

19. A ball is dropped from a height of 2 meters to a floor. On each bounce, the ball rises to 50% of the height of the previous bounce. Calculate the total vertical distance (up and down) the ball travels before coming to rest.

20. Write each series in sigma notation.

a) 5 + 1 + 1/5 + 1/25 + 1/25

b) 2 + 6 + 18 + . . .

c) 3 + 6 + 12 + . . . + 768

16. A student band has two choices of payment to receive for 50 hours of performance.
Choice 1: \$0.10 for the first hour, \$0.12 for the second hour, \$0.144 for the third hour, etc.
Choice 2: \$100 for the first 10 hours, \$200 for the second 10 hours, \$400 for the third 10 hours, etc.
Which set of payments should the students choose to receive?

$S_{50} = \frac{0.10(1-1.2^{50})}{1-1.2} \doteq \$4549.72$

$S_5 = \frac{100(1-2^5)}{1-2} = \$3100$

choice 1

17. If the infinite geometric series has a finite sum, find the sum.

a) 81 + 27 + 9 + 3 + . . .

$r = \frac{1}{3}$ $S = \frac{81}{(1-\frac{1}{3})} = 121.5$

b) 140 + 35 + 8.75 + 2.1875 + . . .

$r = 0.25$ $S = \frac{140}{(1-0.25)} \doteq 186.67$

c) 20 + 30 + 45 + 67.5 + . . .

$r = 1.5$ no finite sum

d) 50 – 25 + 12.5 – 6.25 + . . .

$r = -\frac{1}{2}$ $S = \frac{50}{1-(-\frac{1}{2})} = 33.\bar{3}$

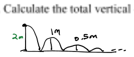
18. Write each series using sigma notation.

a) 40 + 20 + 10 + 5 + 2.5 b) 3 + 9 + 27 + 81 + 243 + 729

$r = \frac{20}{40} = \frac{1}{2}$ $\sum_{n=1}^{5} 40\left(\frac{1}{2}\right)^{n-1}$

$r = \frac{9}{3} = 3$ $\sum_{n=1}^{6} 3(3)^{n-1}$

19. A ball is dropped from a height of 2 meters to a floor. On each bounce, the ball rises to 50% of the height of the previous bounce. Calculate the total vertical distance (up and down) the ball travels before coming to rest.



up: 1 + 0.5 + 0.25 - - .

down: 1 + 0.5 + 0.25 - - .

Sum = drop + ups + downs
= 2 + 2 + 2
= $\boxed{6\text{ m}}$ total vertical distance

$S_{up} = \frac{1}{1-0.5} = 2$ $S_{down} = \frac{1}{1-0.5} = 2$

20. Write each series in sigma notation.

a) 5 + 1 + 1/5 + 1/25 + 1/25

$\sum_{n=1}^{5} 5\left(\frac{1}{5}\right)^{n-1}$

b) 2 + 6 + 18 + . . .

$r = 3$, infinite sum

$\sum_{n=1}^{\infty} 2(3)^{n-1}$

(This will have no finite sum, because $r = 3$, and $-1 < r < 1$ is not met.)

c) 3 + 6 + 12 + . . . + 768

How many terms?
$t_n = 768$, $t_n = ar^{n-1}$
$768 = 3(2)^{n-1}$
$256 = 2^{n-1}$
$2^8 = 2^{n-1}$
$\Rightarrow 8 = n-1$
$n = 9$

Now we know there are 9 terms:

$\sum_{n=1}^{9} 3(2)^{n-1}$

Graph: $f(x) = \frac{x+4}{x^2+5x+4}$

$= \frac{x+4}{(x+1)(x+4)} \longrightarrow$ NPV $= x \neq -1, x \neq -4$

x^2+5x+4

$= \dfrac{x+4}{(x+1)(x+4)}$ → NPV $= x \neq -1,\ x \neq -4$

$f(x) = \dfrac{1}{x+1}$

VA
$x=-1$

HA
$y=0$

POD
@ $x=-4$

$y = \dfrac{1}{-4+1}$

$y = -\frac{1}{3}$

$\left(-4, -\frac{1}{3}\right)$

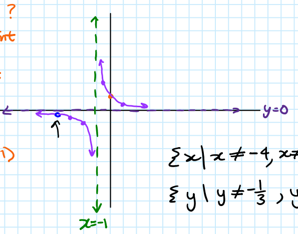
x-int:
$(x+1)\,0 = \dfrac{1}{x+1}$

$0 = 1$?
no x-int

y-int:
$y = \dfrac{1}{0+1}$
$y = 1$
$(0, 1)$

Table

x	y	$\frac{1}{x+1}$
-3	$-\frac{1}{2}$	$\frac{1}{-3+1}$
-2	-1	$\frac{1}{-2+1}$
-0.5	2	$\frac{1}{-0.5+1}$
1	$\frac{1}{2}$	$\frac{1}{1+1}$



$\{x \mid x \neq -4, x \neq -1, x \in \mathbb{R}\}$

$\{y \mid y \neq -\frac{1}{3}, y \neq 0, y \in \mathbb{R}\}$