Class _23 Apr 13 - Applications of Rational Expressions

Tuesday, April 11, 2023 2:28 PM

Tonight's Class:

- **Collecting optional Trig Booster worksheet**
- **Questions from 6.4-6.5?**
- **Working through section 6.6**
  - **Applications of Rational Equations**
- **Work on practice questions from worktext**

$$\frac{n+5}{n+8} = 1 + \frac{6}{n+1}$$

$\left\{-\frac{17}{3}\right\}$

1) NPVs $n \neq -8$, $n \neq -1$

2) LCD : $(n+8)(n+1)$

3) eliminate fractions :

$$(n+8)(n+1)\left[\frac{n+5}{n+8}\right] = (n+8)(n+1)(1) + (n+8)(n+1)\left[\frac{6}{n+1}\right]$$

$$(n+1)(n+5) = (n+8)(n+1) + 6(n+8)$$

4) distribute

$$n^2 + 5n + n + 5 = n^2 + n + 8n + 8 + 6n + 48$$

$$n^2 + 6n + 5 = n^2 + 9n + 8 + 6n + 48$$

$$6n + 5 = 15n + 56$$

$-15n \quad -15n$

$$-9n + 5 = 56$$

$-5 \quad -5$

$$-9n = 51$$

$$\frac{-9n}{-9} = \frac{51}{-9}$$

$$n = \frac{51}{-9} \Rightarrow n = \frac{17}{-3}$$

$$\frac{1}{x^2 - 5x} = \frac{x+7}{x} - 1$$

1) NPVs $x \neq 0$, $x \neq 5$

$$\frac{1}{x^2-5x}=\frac{x+7}{x}-1$$

$\left\{\frac{36}{7}\right\}$

1) NPVs

$x \neq 0$
$x \neq 5$

$x(x-5)\left[\frac{1}{x(x-5)}\right] = x(x-5)\left[\frac{x+7}{x}\right] - [1]\,x(x-5)$

2) LCD $= x(x-5)$

3) Multiply

$1 = (x-5)(x+7) - x(x-5)$

$1 = x^2+7x-5x-35-[x^2-5x]$

$1 = x^2+2x-35-x^2+5x$

$1 = 7x-35$

$+35 \quad +35$

$\frac{7x}{7} = \frac{36}{7}$

$x = \frac{36}{7}$

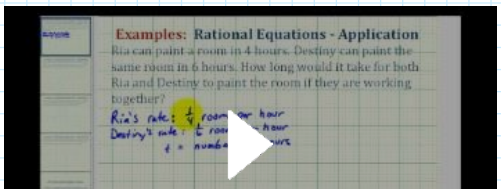
6.6 Applications of Rational Equations

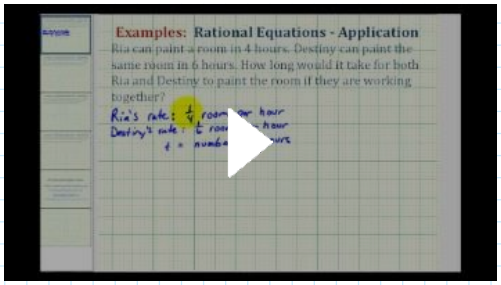
Focus: Solve problems by writing, then solving equations involving rational expressions

Rational equations can be used to solve a variety of problems that involve rates, times and work. Using rational expressions and equations can help you answer questions about how to combine workers or machines to complete a job on schedule.

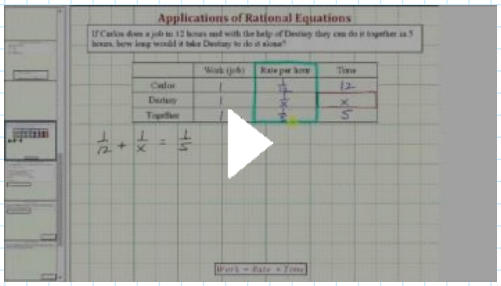
A Good Day's Work

Ex 1: Rational Equation Application - Painting Together

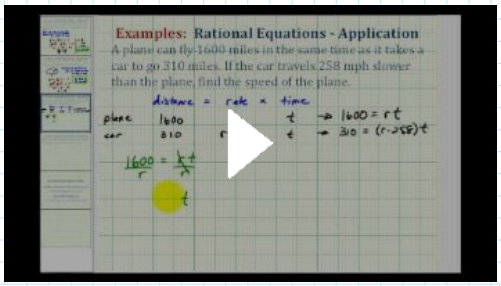




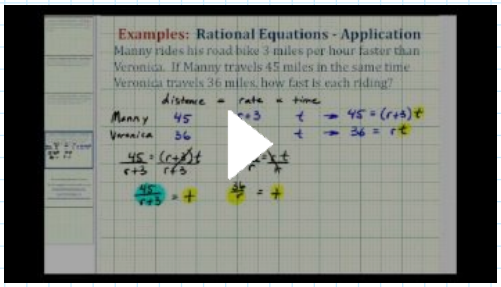
Ex: Rational Equation App - Find Individual Working Time Given Time Working Together



Ex 3: Rational Equation Application - Plane and Car Travelling the Same Time



Ex 4: Rational Equation Application - Two Bikers Riding Different Distances



Motion Problems

Motion problems use the fact that $D = rt$. We will organize the given facts in a table, create an equation and solve it. We often use this rearrangement of the formula:

$$t = \frac{D}{r}$$

$$\frac{D}{r} = \frac{rt}{r}$$

$D = rt$

$t = \frac{D}{r}$

Example

The speed of a car is 5 miles per hour (mph) faster than the speed of a bus. The car travels 220 miles in the same amount of time it takes the bus to travel 200 miles. Find the speed of the car and the speed of the bus.

r = speed of the bus

$r+5$ = speed of the car

	Distance	Rate	Time
car	220	$r+5$	$\frac{220}{r+5}$
bus	200	r	$\frac{200}{r}$

equation: $r(r+5)\left[\frac{220}{r+5}\right] = \left[\frac{200}{r}\right] r(r+5)$

1) NPVs:

$r \neq -5$

$r \neq 0$

2) LCD

$r(r+5)$

$220r = 200(r+5)$

$220r = 200r + 1000$

$-200r \quad -200r$

$\frac{20r}{20} = \frac{1000}{20}$

$r = 50$

Bus - 50 mph

Car - 55 mph

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Example 1 **Solving Problems Involving Motion**

Jerome rows his boat 24 km downstream and back to where he began. When the average speed of the current is 2 km/h, Jerome can complete the journey in 9 h. What is Jerome's average rowing speed in still water?

r = Jerome's speed in still water

	D	R	T
downstream (current helps you)	24	$r+2$	$\frac{24}{r+2}$
upstream (current slows you down)	24	$r-2$	$\frac{24}{r-2}$

$D = rt$, we know $t = \frac{D}{r}$

upstream (current slows you down)

know $t = \frac{d}{r}$

downstream time + upstream time = total time 9

$$\frac{24}{r+2} + \frac{24}{r-2} = 9$$

$$(r+2)(r-2)\left[\frac{24}{r+2}\right] + \left[\frac{24}{r-2}\right](r+2)(r-2) = [9](r+2)(r-2)$$

1) NPVs

$r \neq 2$

$r \neq -2$

2) LCD

$(r+2)(r-2)$

$$24(r-2) + 24(r+2) = 9(r+2)(r-2)$$

$$24r - 48 + 24r + 48 = 9(r^2 - 2r + 2r - 4)$$

$$48r = 9(r^2 - 4)$$

$$48r = 9r^2 - 36$$

$$0 = 9r^2 - 48r - 36$$

$$\frac{0}{3} = \frac{3}{3}(3r^2 - 16r - 12)$$

Collect all terms on one side of equation and factor, if possible. If it doesn't factor, use the quadratic formula.

$$3r^2 - 16r - 12 = 0$$

AC = $3(-12) = -36$

add to -16: -18 and 2

$$3r^2 - 18r + 2r - 12 = 0$$

$$3r(r-6) + 2(r-6) = 0$$

$$(3r+2)(r-6) = 0$$

$3r + 2 = 0$

$3r = -2$

$r = -\frac{2}{3}$ — eliminate this. Because it doesn't make sense for a rowing speed

$r - 6 = 0$

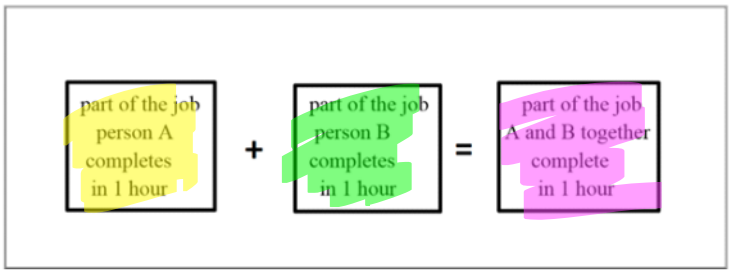
$r = 6$

Jerome - 6 km/h in still water

Motion Questions: p 591, CYU #1, page 595: #4, 8, 11, 12, 13

Work Problems

Work problems often ask you to calculate how long it will take different people working at different speeds to finish a task.



this is a bit different than how the worktext does it

Example

David can paint a house in 12 hours. Bill can paint the same house in 9 hours. How long would it take them to paint the house together?

t = amount of time for them to paint the whole house together

	time for whole job	how much of the job is done in 1 hour
David	12 hours	$\frac{1}{12}$ of it
Bill	9 hours	$\frac{1}{9}$ of it
together	t hours	$\frac{1}{t}$ of it

$$\overset{3}{\cancel{36}t}\left[\frac{1}{\cancel{12}_{1}}\right] + \overset{4}{\cancel{36}t}\left[\frac{1}{\cancel{9}_{1}}\right] = \cancel{36}t\left[\frac{1}{\cancel{t}}\right]$$

NPVs $t \neq 0$

$$3t + 4t = 36$$

$$\frac{7t}{7} = \frac{36}{7}$$

$$t = \frac{36}{7} \text{ hours}$$

$$= 5\frac{1}{7} \text{ hours}$$

$(\doteq 5.1$ hours only if it tells us to round$)$

LCD

$12 = 2 \cdot 2 \cdot 3$

$9 = 3 \cdot 3$

$t = t$

$2 \cdot 2 \cdot 3 \cdot 3t$

LCD $= 36t$

WT, page 592

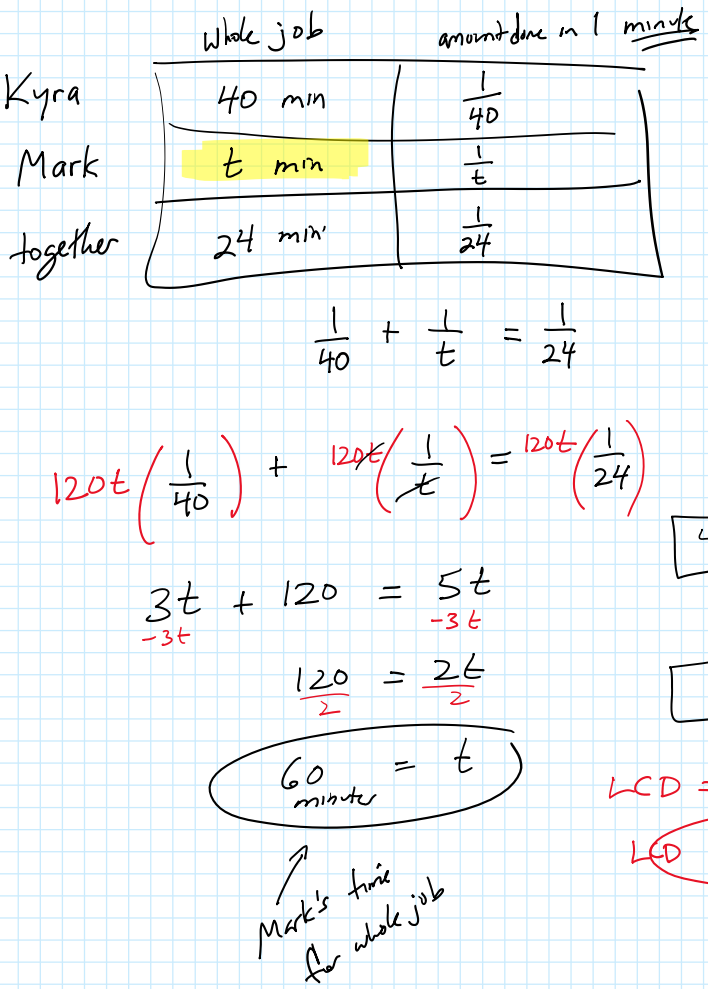
Example 2 **Solving Problems Involving Work**

Kyra mows a lawn in 40 min. When Mark and Kyra work together, they can mow the lawn in 24 min. How long would it take Mark to mow the lawn on his own?

(minutes)

t = Mark's time for whole job

Whole job	amount done in 1 minute



Work Questions: p 592, CYU #2, page 595: #3, 5, 6, 10

Proportion Problems

WT, page 594

Example 3 Solving Problems Involving Proportion

Antifreeze is added to water to make a solution that is used in automobiles. How much antifreeze must be added to 12 L of water to make a solution that contains 40% antifreeze?

Water + antifreeze = solution

12 L A 40% antifreeze

volume of desired quantity → $\frac{A}{12+A} = \frac{40}{100}$ ← total volume

NPVs: $A \neq -12$

LCD: $100(12+A)$

$100(12+A)\left[\frac{A}{12+A}\right] = \left[\frac{40}{100}\right]100(12+A)$

$100A = 40(12+A)$

$100A = 480 + 40A$

$-40A \quad -40A$

$\frac{60A}{60} = \frac{480}{60}$

$A = 8$ L of antifreeze

Check: $\frac{A}{12+A} = \frac{8}{12+8} = \frac{8}{20} = 0.4$ (40%)

Proportion Questions: p 594, CYU #3, page 597: #7

For next class

- **Complete worktext questions for 6.6**
- **Complete the Chapter 6 Hand-in, due Tuesday, April 18**
- **Prepare for the Chapter 6 Test, on Tuesday, April 18**
- **Work on preparing for the Unit 3 Exam, on Thursday, April 20**

Timeline:

- **Tuesday, April 18 - Chapter 6 Test, sections 7.1-7.2**
- **Thursday, April 20 - Unit 3 Exam (Chapters 5 and 6)**
- **Tuesday, April 25 - sections 7.2-7.3**
- **Thursday, April 27 - Chapter 7 Test. Last class**