Factoring and Equation-Solving Review

Factoring the Difference of Squares

Whenever factoring, we check to see if the expression is a ***difference of squares***. Remember that in math:

difference – means to subtract

squares – are numbers or expressions that result from multiplying something with itself

Some numbers that are called **perfect squares** are 9, 16, and 64.

$(3)^2 = 9$ $\quad\quad$ $(8)^2 = 64$ $\quad\quad$ $(4)^2 = 16$

Expressions like x^2, w^4, and $81x^2y^2$ are also called perfect squares.

$x^2 = (x)(x)$ $\quad\quad$ $w^4 = (w^2)(w^2)$ $\quad\quad$ $81x^2y^2 = (9xy)(9xy)$

To factor a difference of squares, take the square root of each term. Write one factor as the sum of the square roots and the other factor as the difference of the square roots.

$$a^2 - b^2 = (a + b)(a - b)$$

Example 1

Factor completely. $81x^2 - 25$

1) Check for a GCF. There is none.

2) Notice this is a "difference." Is each term a perfect square? Yes, so we factor it using the pattern shown in the box above.

$$81x^2 - 25 = (9x - 5)(9x + 5)$$

3) Check work by multiplying and seeing whether the result is in fact the original expression.

$$(9x - 5)(9x + 5) \quad = 81x^2 + 45x - 45x - 25$$

$$= 81x^2 - 25$$

Example 2

Factor completely. $1 - 16y^2$

1) Check for a GCF. There is none.

2) Notice this is a "difference." Is each term a perfect square? Yes.

$$1 - 16y^2 = (1 - 4y)(1 + 4y)$$

3) Check work by multiplying and seeing whether the result is the original expression.

$$\begin{aligned}(1 - 4y)(1 + 4y) \quad &= 1 - 4y + 4y + 16y^2 \\ &= 1 - 16y^2\end{aligned}$$

Example 3

Factor completely. $2x^4 - 32$

1) Check for a GCF. The number "2" is a common factor; factor it out.

$$2x^4 - 32 \quad = 2(x^4 - 16)$$

2) Notice the expression in the bracket is a difference of squares. Factor it.

$$= 2(x^2 - 4)(x^2 + 4)$$

3) The first binomial is itself a difference of squares and so also must be broken down into its factors.

$$= 2(x - 2)(x + 2)(x^2 + 4)$$

Factoring Trinomials

A trinomial is an algebraic expression consisting of 3 terms. Ones in the form $ax^2 + bx + c$ will often factor. If they do factor, they factor into two binomial factors.

Example 1

Factor completely. $x^2 + 7x + 12$

1) Check for a GCF. There is none.

2) Write down 2 empty brackets, each big enough for a binomial. In the "first" spot in each bracket, place the needed variable, so that when you multiply the first term in each binomial with each other, you will get the first term of the trinomial.

$$(x \qquad)(x \qquad)$$

3) Now, think of two numbers that
   - multiply to give the constant value (12)
   - add to give the value of the linear coefficient (7). ***(3 and 4 are the numbers)***

4) Write these numbers in the brackets. $(x + 3)(x + 4)$

5) Check by multiplying. $(x + 3)(x + 4) = x^2 + 3x + 4x + 12 = x^2 + 7x + 12$

Example 2

Factor completely. $3x^2 - 6x - 105$

1) Check for a GCF: $3x^2 - 6x - 105 = \quad 3(x^2 - 2x - 35)$

2) Write down 2 empty brackets, each big enough for a binomial. In the "first" spot in each bracket, place the needed variable, so that when you multiply the first term in each binomial with each other, you will get the first term of the trinomial.

$$3(x \qquad)(x \qquad)$$

3) Now, think of two numbers that
   - multiply to give the constant value (-35)
   - add to give the value of the linear coefficient (-2). ***(-7 and 5 are the numbers)***

4) Write these numbers in the brackets. $3(x - 7)(x + 5)$

5) Check by multiplying. $3(x - 7)(x + 5) = 3(x^2 + 5x - 7x - 35)$

$$= 3(x^2 - 2x - 35)$$

$$= 3x^2 - 6x - 105$$

Some trinomials have a leading coefficient that is not 1. They can be factored using several methods. Here are two examples, one showing the "***guess and check***" method and the other the ***decomposition or AC*** method. You may have learned a different method. So long as you can clearly show your work and consistently get the correct answers, you may use whichever method you wish.

Example 3

Factor completely, using "guess and check." $2x^2 - x - 21$

1) Check for a GCF. There is none.

2) Write down 2 empty brackets, each big enough for a binomial. In the "first" spot in each bracket, place the needed expression, so that when you multiply the first term in each binomial with each other, you get the first term of the trinomial. $(2x \quad\quad)(x \quad\quad)$

3) Make a list of the different possibilities for two numbers that multiply to give the constant value (-21)

-3, 7   3, -7   1, -21   -1, 21

4) Select one of the possibilities and write it in the brackets. Multiply to see if this choice gives the correct middle term of the original trinomial. Realize that putting the numbers in different locations will create different trinomial.

$(2x - 3)(x + 7) = 2x^2 + 14x - 3x - 21$ middle term is $11x$, not what we need

$(2x + 7)(x - 3) = 2x^2 - 6x + 7x - 21$ middle term is x, not what we need

$(2x - 7)(x + 3) = 2x^2 + 6x - 7x - 21$ middle term is $-x$, which ***is*** what we need

5) Final answer: $2x^2 - x - 21 = (2x - 7)(x + 3)$

This method works best when the leading coefficient and the constant term are prime numbers.

Keep track of what combinations you have tried so you don't accidentally keep trying the same ones over again!

Example 4

Factor completely, using decomposition, or the “AC” method. $3x^2 - 2x - 5$

1) identify A, B, and C : A = 3, B = -2, C = -5

2) calculate “AC” AC = (3)(-5) = -15

3) Find two numbers that multiply to give AC (-15) and add to give B (-2). ***-5 and 3***

4) Re-write the trinomial as a four-term expression. Split (or “decompose) the linear term into 2 terms whose coefficients are the numbers we just found in the previous step.

$$3x^2 - 2x - 5 \quad = 3x^2 \mathbf{- 5x + 3x} - 5$$

5) Factor out the GCF of the first two terms, then the GCF of the last two terms, as shown below:

$$3x^2 - 5x + 3x - 5 \quad = x(3x - 5) + 1(3x - 5)$$

6) Factor out the common binomial as a GCF: $(3x - 5)(x + 1)$

7) Check your answer by expanding.

$$(3x - 5)(x + 1) = 3x^2 + 3x - 5x - 5$$

$$= 3x^2 - 2x - 5$$

Solving Equations

1) Linear Equations

"Linear" means that if the equation is graphed, you get a straight line. Variables in linear equations are not raised to any exponent other than 1.

Here are some steps to guide you when solving linear equations:

1) If there are brackets, distribute.

2) If there are fractions, eliminate them by multiplying each term by the least common denominator.

3) Collect all the terms containing the variable on one side of the equation and all terms that are constants on the other side of the equation.

4) Combine all like terms.

5) If the variable has a coefficient other than 1, divide to eliminate it.

Example 1

Solve for x.

$$3(x-5) = 21$$

$$3x - 15 = 21$$

$$3x - 15 + 15 = 21 + 15$$

$$3x = 36$$

$$x = 12$$

Example 2

Solve for *x*.

$$\frac{3x}{8}+\frac{2}{5}=-2$$

$$40\left(\frac{3x}{8}+\frac{2}{5}\right)=40(-2)$$

$$\frac{40}{1}\left(\frac{3x}{8}\right)+\frac{40}{1}\left(\frac{2}{5}\right)=\frac{40}{1}\left(\frac{-2}{1}\right)$$

$$\frac{\overset{5}{\cancel{40}}}{1}\left(\frac{3x}{\underset{1}{\cancel{8}}}\right)+\frac{\overset{8}{\cancel{40}}}{1}\left(\frac{2}{\underset{1}{\cancel{5}}}\right)=\frac{40}{1}\left(\frac{-2}{1}\right)$$

$$5(3x)+8(2)=40(-2)$$

$$15x+16=-80$$

$$15x+16-16=-80-16$$

$$15x=-96$$

$$x=-\frac{96}{15}$$

2) Quadratic Equations

"Quadratic" means that if the equation is graphed, you get a parabola. The highest exponent to which a variable in a quadratic equation can be raised is the exponent 2.

To solve quadratic equations:

1) Do whatever algebra is required to get all terms on one side of the equation, and "0" on the other side of the equation.

2) Factor the expression you now see in your equation.
   a) Set each factor, one at a time, equal to "0."
   b) Solve each of these linear equations.

3) If the expression does NOT factor, use the quadratic formula to solve.

Example 1

Solve for *x*.

$$x^2 = 40 - 3x$$

$$x^2 + 3x = 40 - 3x + 3x$$

$$x^2 + 3x - 40 = 40 - 40$$

$$x^2 + 3x - 40 = 0$$

$$(x-5)(x+8) = 0$$

$$x - 5 = 0, \quad x = 5 \qquad x + 8 = 0, \quad x = -8$$

Solutions: $x = 5, \; x = -8$

Example 2

Solve for x. $$x^2 + 9x + 5 = 0$$

Unfortunately, there are no numbers that multiply to 5 and also add to 9. This means we cannot factor this expression. However, we ***can*** solve it by using the quadratic formula.

> The quadratic formula is used to solve any quadratic equation in the form $ax^2 + bx + c = 0$. The solutions are found by substituting the values of a, b, and c into this formula:
>
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The equation is $x^2 + 9x + 5 = 0$, so we substitute into the formula the values a = 1, b = 9, c = 5:

$$x = \frac{-9 \pm \sqrt{(9)^2 - (4)(1)(5)}}{2(1)}$$

This gives us two solutions: $x = \frac{-9 + \sqrt{61}}{2}$, and $x = \frac{-9 - \sqrt{61}}{2}$

Decimal approximations to the solutions are:

$x \approx -0.59$, and $x \approx -8.41$

Practice Questions

Complete all questions. Show all your work clearly. Check your answers using the key given at the end.

Factoring:

Factor each of the following expressions completely.

1. $x^2 - 46x + 45 =$

2. $x^2 + 6x - 16 =$

3. $w^2 - 7wx - 44x^2 =$

4. $2x^2 + 12x + 10 =$

5. $-10x^2 + 20x + 150 =$

6. $3x^2 + 8x - 11 =$

7. $2x^2 + 3x + 1 =$

8. $9x^2 - 16y^2 =$

Solving Linear and Quadratic Equations:

Solve each of the following equations. Clearly show the solution process. If answer is a fraction, leave it in fractional form.

1. $4(m-1)-6m=-10(2m-1)-1$

2. $\dfrac{5m+2}{2}=\dfrac{3m-1}{3}$

3. $\dfrac{m}{4}+5m=\dfrac{1}{2}m+2$

4. $3x^2+16x+5=0$

5. $6x^2+17x=-5$

6. $x^2+3=5x$

7. $-4x^2-11x-6=0$

8. $x^2-100=0$

Answers

Factoring:

1. (x-45)(x-1)
2. (x+8)(x-2)
3. (w-11x)(w+4x)
4. 2(x+5)(x+1)
5. -10(x-5)(x+3)
6. (3x+11)(x-1)
7. (2x+1)(x+1)
8. (3x-4y)(3x+4y)

Solving:

1. $\frac{13}{18}$
2. $-\frac{8}{9}$
3. $\frac{8}{19}$
4. $x = -5,\ x = -\frac{1}{3}$
5. $x = -\frac{1}{3}, x = -\frac{5}{2}$
6. Exact answers: $x = \frac{5 \pm \sqrt{13}}{2}$; approximations: $x \doteq 4.30,\ x \doteq 0.70$
7. $x = -\frac{3}{4}, x = -2$
8. $x = -10, x = 10$