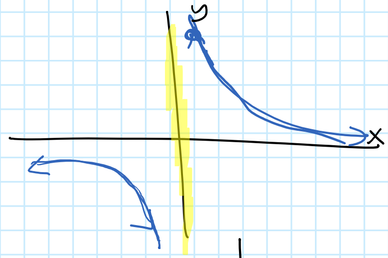
Annalyn 9.2 Notes

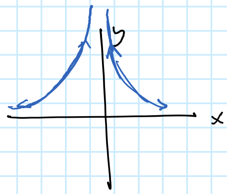
Saturday, April 2, 2022 8:03 PM

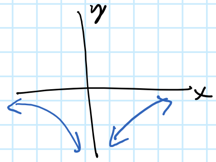
$y = \frac{1}{x}$

$y = \frac{a}{x-h} + k$



$y = \frac{a}{x^2}$





9.2 Analyzing Rational Functions

Some rational function equations are more complicated. To analyze and graph them, we *factor and simplify* their equations.

Example

Consider the rational function: $f(x) = \dfrac{x^2 + 7x + 12}{x + 4}$ NPV $x = -4$

a) Factor and simplify the function's equation

$f(x) = \dfrac{(x+3)(x+4)}{x+4}$ $x=-4$ NPV

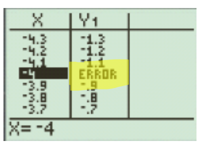
simplify

$f(x) = x + 3$, $\underbrace{\text{where } x \neq -4}_{\text{include this}}$

b) NPV(non-permissible value) = -4

How does the graph behave near its NPV?

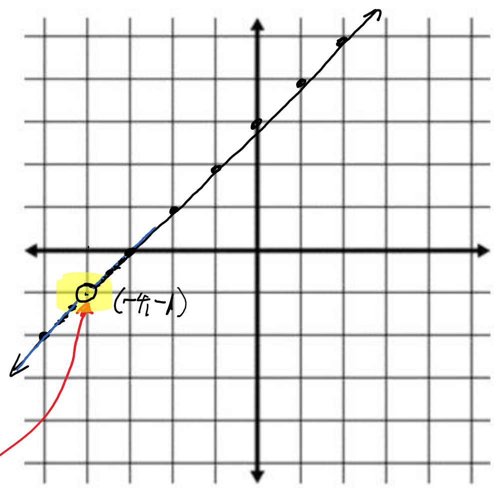
very smoothly. Not like the behavior near NPVs that create asymptotes.



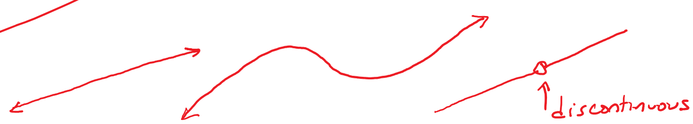
$f(x) = x+3$

$y = x + 3$

x	$f(x)$
-5	-2
-4	does not exist! DNE
-3	0
-2	1
-1	2
0	3
1	4
2	5



POD
$(-4,-1)$



Point of Discontinuity (POD) – an ordered pair where the graph of a function does not exist. It occurs whenever the equation's numerator and denominator have a common factor that includes a variable.

(x, y)

$f(x) = x + 3$

plug in the NPV value, $x=-4$, into this:

$= -4 + 3 = -1$

To get the y-value of a POD, we substitute the x-value (the NPV) into the reduced, simplified version of the function's equation.

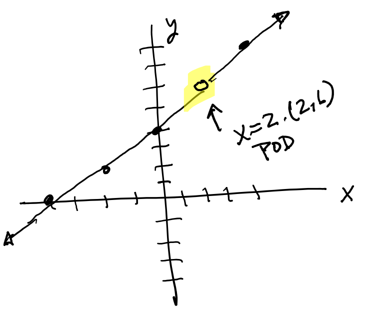
Example

a) Complete the table, with the characteristics of the two graphs.

	$y = f(x) = \frac{x^2+2x-8}{x-2}$	$g(x) = \frac{x^2+6x+8}{x-2}$
Non-permissible value(s) NPV	$x = 2$	$x = 2$
Simplified form of equation	$f(x) = \frac{\cancel{(x-2)}(x+4)}{\cancel{x-2}}$ $f(x) = x+4$, where $x \neq 2$	$g(x) = \frac{(x+2)(x+4)}{x-2}$, where $x \neq 2$
Coordinates of x- and y-intercepts	x-int: $y = x+4$, $0 = x+4$, $x = -4$, $(-4, 0)$ y-int: $y = 0+4$, $y = 4$, $(0, 4)$	x-int: $y = \frac{(x+2)(x+4)}{x-2}$ $0 = \frac{(x+2)(x+4)}{x-2} \cdot (x-2)$ $0 = (x+2)(x+4)$ $x+2=0$, $x=-2$, $(-2, 0)$ $x+4=0$, $x=-4$, $(-4, 0)$ y-int: $y = \frac{(x+2)(x+4)}{x-2}$ $y = \frac{(0+2)(0+4)}{0-2}$ $y = \frac{8}{-2} = -4$ $(0, -4)$

b) Graph these rational functions (same as the ones above) using technology. Below each equation draw a rough sketch of its graph.

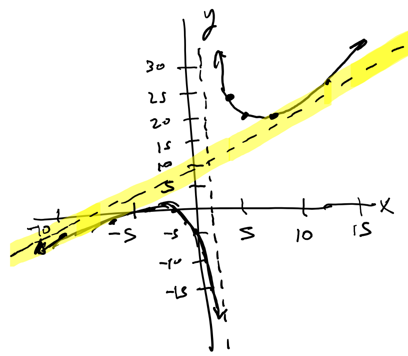
$f(x) = \frac{x^2+2x-8}{x-2}$, $f(x) = x+4$, where $x \neq 2$



x	y
-4	0
-2	2
0	4
2	DNE
4	8

POD $y = x+4$
$= 2+4$
$= 6$
$(2, 6)$

$g(x) = \frac{x^2+6x+8}{x-2} = \frac{(x+2)(x+4)}{x-2}$



x = 2
v.a.

x	y
-10	-4
-6	-1
-4	0
-3	0.2
-2	0
-1	-1
0	-4
1	-15
4	24
6	20
8	20
14	24

When does a rational function have
- ***a point of discontinuity*** — whenever a factor on the top (that has x in it) cancels with a factor on the bottom
- ***a vertical asymptote?*** — whenever there's an NPV in a factor that does NOT cancel.

Whiteboards - PODs and horizontal asymptotes

Whiteboards - <u>PODs and horizontal asymptotes</u>

- **Horizontal asymptotes questions #1 and #2, we can get the h.a. equations from remembering the two base graphs we learned**
- **Horizontal asymptotes questions #3-6, these equations are not in the form of the base graphs, but we can get the h.a. equations from looking at the graphs**
- **Horizontal asymptotes questions #7 - how can we get the h.a. equation when the original function equations are more unusual?**

Key Ideas for Rational Function Graphs

1) Horizontal Asymptotes

Find the degree of the numerator and denominator.

	example
Numerator degree < Denominator degree ***horizontal asymptote equation:*** $y = 0$	$y = \frac{x^1}{3x^2+4}$ h.a. $y=0$
Numerator degree = Denominator degree ***horizontal asymptote equation:*** $y = \frac{\text{leading coefficient of num}}{\text{leading coefficient of denom}}$	$y = \frac{\boxed{5}x^2-7}{\boxed{2}x^2+3}$ h.a $y = \frac{5}{2}$
Numerator degree > Denominator degree Graph will have a ***slant asymptote***	$y = \frac{4x^2-9}{2x^1+3}$ slant asymptote

2) NPVs, PODs, and vertical asymptotes

Factor numerator and denominator completely.

- Set ***each factor of the denominator = 0***, to get all NPVs.
    - Is there a factor that cancels with a factor in the numerator? It gives the x-value of a POD.
    - Is there a factor that doesn't cancel with a numerator factor? It gives the location of a vertical asymptote.

3) Intercepts

- ***y*-intercepts** – substitute $x = 0$ into the function (either the original or the simplified form) and solve for y
- ***x*-intercepts** - set each factor of the $\underbrace{\text{simplified numerator} = 0 \text{ and solve for } x}$

  let $y=0$ and solve for x

4) Sketch

- Plot all x-intercepts and y-intercepts
- Show points of discontinuity (PODs) as "holes", using an open circle
- Show all asymptotes as dotted lines.
- Find more points on the graph, as needed, by substituting into its equation.
- Make sure graph does not cross any vertical asymptotes.

To try:

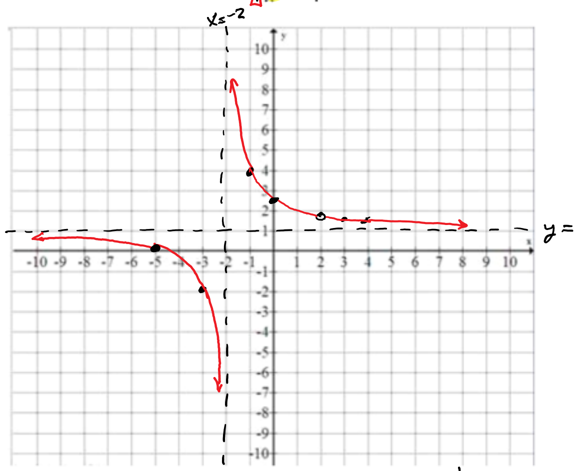
Original Equation	Factored form of equation	List all NPVs, and for each one identify if it gives a POD or a vertical asymptote. - Find the (*x, y)* coordinates of each POD . - Find the equation of each vertical asymptote.	Horizontal asymptote equation or say "Slant"
$y=\frac{2x^1+10}{x^2+2x-15}$	$y=\frac{2\cancel{(x+5)}}{(x-3)\cancel{(x+5)}}$ $y=\frac{2}{x-3}$, where $x\neq -5$	$x=3$ v.a. $x=-5$ POD plug in $x=-5$: $y=\frac{2}{x-3}$ $y=\frac{2}{-5-3}$ $y=\frac{2}{-8}$ $y=-1/4$ $\boxed{\left(-5, -\frac{1}{4}\right)}$	$\frac{\text{degree } 1}{\text{degree } 2}$ $\Rightarrow$ $\boxed{y=0}$
$y=\frac{2x^2+7x+6}{1x^2-2x-8}$	$y=\frac{(2x+3)\cancel{(x+2)}}{\cancel{(x+2)}(x-4)}$ $y=\frac{2x+3}{x-4}$, where $x\neq -2$	$x=4$ v.a. $x=-2$ POD $y=\frac{2x+3}{x-4}$ $=\frac{2(-2)+3}{-2-4}$ $=\frac{-1}{-6}=+\frac{1}{6}$ $\left(-2, \frac{1}{6}\right)$	$\frac{\text{degree } 2}{\text{degree } 2}$ h.a. $y=\frac{2}{1}$ $\boxed{y=2}$
$y=\frac{x^2+3x-4}{x-1}$	$y=\frac{\cancel{(x-1)}(x+4)}{\cancel{x-1}}$ $y=x+4$, where $x\neq 1$	POD when $x=1$ $y=x+4$ $=1+4$ $=5$ $(1,5)$	$\frac{\text{degree } 2}{\text{degree } 1}$ $\Rightarrow$ $\boxed{\text{slant asymptote}}$

Whiteboards -Characteristics of Rational Functions

Without using technology, accurately sketch the function's graph: $y = \frac{\boxed{1}x^2 + 3x - 10}{\boxed{1}x^2 - 4}$

Give the values of the graph's:

- NPVs
- asymptote equations
- coordinates of PODs
- x- and y-intercepts



$y = \frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}(x+2)}$, where $x \neq 2$

NPVS: $x = -2$ v.a.

$x = 2$ POD

$y = \frac{x+5}{x+2} \Rightarrow y = \frac{2+5}{2+2} = \frac{7}{4}$

$(2, \frac{7}{4})$

hor. asymp: (degree same) $y = \frac{1}{1}$, $\boxed{y = 1}$

x-int $\quad y = \frac{x+5}{x+2} \quad (x+2)\,0 = \frac{x+5}{\cancel{x+2}}\,\cancel{(x+2)}$

$0 = x+5$

$x = -5 \qquad (-5, 0)$

y-int (let $x=0$) $\quad y = \frac{0+5}{0+2}$

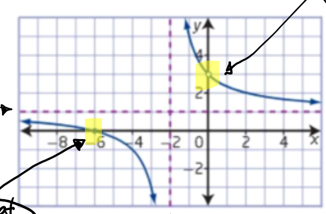
$y = \frac{5}{2} \qquad (0, \frac{5}{2})$

$\boxed{y = \frac{x+5}{x+2}}$

x	y
-5	0
-3	$\frac{-3+5}{-3+2} = \frac{2}{-1} = -2$
-1	$\frac{-1+5}{-1+2} = \frac{4}{1} = 4$
3	$\frac{3+5}{3+2} = \frac{8}{5} = 1\frac{3}{5}$
4	$\frac{4+5}{4+2} = \frac{9}{6} = \frac{3}{2}$

Example (TB page 453, #7a)
Write the equation of the pictured rational function.

h.a. $y=1$



POD, when $x=0$ so, we have $\frac{x-0}{x-0} = \frac{x}{x}$

x-intercept at $x=-6$, so we have $x+6$ on the top

v.a. $x=-2$, so $x+2$ on the bottom

$y = \frac{x(x+6)}{x(x+2)}$

$\boxed{y = \frac{x^2+6x}{x^2+2x}}$

h.a. $y = \frac{1}{1}$, $y=1$

(9.2) TB p 452: 4-7, 8ac, 11, 14