Part 1: Working with Fractions

In this course you need to know how to work with fractions without relying on a calculator.

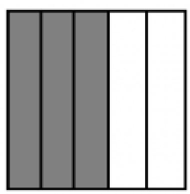
Adding and Subtracting Fractions

Recall from earlier math classes that a fraction is made up of two parts. The bottom part, called the ***denominator***, tells you how many parts the whole is divided into. The top part, called the ***numerator***, tells you how many of those parts you have.

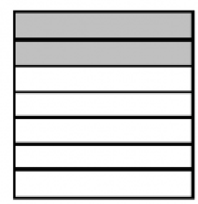
Let's figure this out: $\frac{3}{5}+\frac{2}{7}=$

Many people have forgotten how to do that and so either reach for their calculators, getting a very ugly decimal that they round to 0.89, or they guess – maybe coming up with $\frac{5}{12}$.

$\frac{3}{5}$ means 3 out of 5 parts

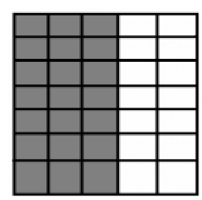


and $\frac{2}{7}$ means 2 out of 7 parts.

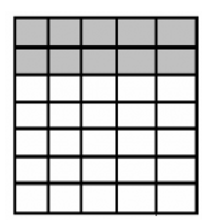


Before we can add these together, we need the pieces we are adding to be the ***same size***. The shaded rectangles in the $\frac{3}{5}$ diagram are NOT the same size as the ones in the $\frac{2}{7}$ diagram.

At right we still have shaded $\frac{3}{5}$ of the whole box, but by subdividing it into seven rows, we see that the shaded amount is also equal to $\frac{21}{35}$ of the box.



In the box at the right, the portion that is shaded is $\frac{2}{7}$. We subdivide this box into five columns and find that the shaded amount is also equal to $\frac{10}{35}$.



$$\frac{3}{5}+\frac{2}{7} = \qquad \frac{21}{35}+\frac{10}{35} = \qquad \frac{31}{35}$$

Example 1

Evaluate each of the following.

a) $\frac{9}{4}-\frac{2}{3}$

Solution

$$\frac{9}{4}-\frac{2}{3} = \left(\frac{9}{4}\right)\cdot\left(\frac{3}{3}\right)-\left(\frac{2}{3}\right)\cdot\left(\frac{4}{4}\right)$$

Multiplying by $\frac{3}{3}$ or by $\frac{4}{4}$ is like multiplying by "1."

$$= \frac{27}{12}-\frac{8}{12}$$

$$= \frac{19}{12}$$

Doing this does not change the value of the original fractions, but it makes them have a ***common denominator*** so they can be combined.

This answer can also be written as a mixed fraction: $1\frac{7}{12}$

b) $\frac{\pi}{4}+2\pi$

Solution

$$\frac{\pi}{4}+2\pi = \frac{\pi}{4}+\frac{2\pi}{1}$$

When one of the terms is not written as a fraction, we make it look like one by putting "1" underneath it as its denominator.

$$= \frac{\pi}{4}+\left(\frac{2\pi}{1}\right)\cdot\left(\frac{4}{4}\right)$$

Get a common denominator.

$$= \frac{\pi}{4}+\frac{8\pi}{4}$$

Add.

$$= \frac{9\pi}{4}$$

Fractions with π in them are not usually written in mixed form.

Multiplying and Dividing Fractions

Multiplying fractions and dividing fractions are both easier processes than adding fractions or subtracting fractions. The reason is that **<u>we do NOT need to get a common denominator when multiplying or dividing fractions</u>**.

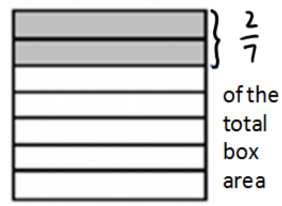
Let's figure this out: $\frac{3}{5} \times \frac{2}{7} =$

You may remember that all we need to do is multiply directly across. The answer is: $\frac{6}{35}$.

Why does this method work?

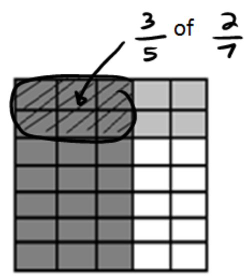
$\frac{3}{5} \times \frac{2}{7}$ means $\frac{3}{5}$ of the region with a size that is $\frac{2}{7}$ of the size of the entire box.

The shaded portion at right is $\frac{2}{7}$ of the entire box.



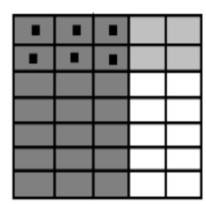
Now we split the entire box into 5 equal pieces (up-and-down columns), and shade 3 of those 5 pieces.

The overlap of the two shaded areas, circled at right, is $\frac{3}{5}$ of $\frac{2}{7}$.



How many of the 35 little boxes are in the overlap?

6 of them, so the answer is $\frac{6}{35}$



What about this question? $\frac{3}{5} \div \frac{2}{7} =$

We change this to a multiplication question by ***inverting the second fraction***:

$\frac{3}{5} \times \frac{7}{2} = \frac{21}{10}$, or $2\frac{1}{10}$

Example 2

Evaluate each of the following.

a) $\frac{21}{4}\times\frac{2}{7}$

Solution

$$\frac{21}{4}\times\frac{2}{7} = \frac{21}{4}\times\frac{2}{7}$$

We can multiply directly across and then reduce. Sometimes this gives us quite large numbers to reduce.

$$= \frac{7\times 3}{2\times 2}\times\frac{2}{7}$$

A way to avoid large numbers is to ***reduce first,*** before multiplying. We factor first and then cancel identical factors to reduce.

$$= \frac{\cancel{7}\times 3}{2\times \cancel{2}}\times\frac{\cancel{2}}{\cancel{7}}$$

$$= \frac{3}{2}$$

This method is also useful for working with rational expressions involving variables.

b) $\frac{x^2-81}{4x}\div\frac{x-9}{11x^2}$

Solution

$$\frac{x^2-81}{4x}\div\frac{x-9}{11x^2} = \frac{x^2-81}{4x}\times\frac{11x^2}{x-9}$$

Invert the second fraction at the first step.

$$= \frac{(x+9)\cancel{(x-9)}}{4\cancel{x}}\times\frac{(11)\cancel{(x)}(x)}{\cancel{x-9}}$$

Factor completely. ***(See the next section for a factoring review!)***

$$= \frac{(x+9)}{4}\times\frac{(11)(x)}{1}$$

Reduce. A factor on the top divided by an identical factor on the bottom can be cancelled, since it is simply equal to "1."

$$= \frac{11x(x+9)}{4}$$

Part 2: Factoring

To factor a number means to write it as a product of two or more numbers.

15 can be factored as 5 x 3. The three and the five here are called the ***factors*** of 15.

To factor completely means that each factor must be as simple as possible. In other words, the factors themselves cannot be factored.

24 = 8 x 3	this is ***not*** factored completely, since the 8 itself can be factored
24 = 2 x 2 x 2 x 3	this ***is*** factored completely

To factor an algebraic expression means to write it as a product of two or more algebraic factors.

We'll review 3 types of factoring:

1) Greatest Common Factor (GCF)

2) Difference of Squares

3) Trinomial Factoring

1) Greatest Common Factor (GCF)

In all factoring questions, check first to see if there is a factor that divides evenly into each of the terms in the algebraic expression. If there is, factor it out in front!

Example 1

Factor completely. $9x^2y^4 + 18x^3y^2 + 15xy^2$

$3xy^2$ is the term with the largest coefficient and highest exponents that divides evenly into each of the three terms in this expression. This means it is the greatest common factor and we will factor out in front:

$$9x^2y^4 + 18x^3y^2 + 15xy^2 = 3xy^2\left(3xy^2 + 6x^2 + 5\right)$$

Example 2

Factor completely. $10w^3 + 5w - 15w^2$

The greatest common factor (GCF) this time is 5w.

$$10w^3 + 5w - 15w^2 = 5w(2w^2 + 1 - 3w)$$

2) Factoring the Difference of Squares

Whenever factoring, we check to see if the expression is a ***difference of squares***. Remember that in math:

difference – means to subtract

squares – are numbers or expressions that result from multiplying something with itself

Some numbers that are called **perfect squares** are 9, 16, and 64.

$(3)^2 = 9$ $\quad\quad$ $(8)^2 = 64$ $\quad\quad$ $(4)^2 = 16$

Expressions like x^2, w^4, and $81x^2y^2$ are also called perfect squares.

$x^2 = (x)(x)$ $\quad\quad$ $w^4 = (w^2)(w^2)$ $\quad\quad$ $81x^2y^2 = (9xy)(9xy)$

To factor a difference of squares, take the square root of each term. Write one factor as the sum of the square roots and the other factor as the difference of the square roots.

$$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$$

Example 1

Factor completely. $81x^2 - 25$

1) Check for a GCF. There is none.

2) Notice this is a "difference." Is each term a perfect square? Yes, so we factor it using the pattern shown in the box above.

$$81x^2 - 25 = (9x - 5)(9x + 5)$$

3) Check work by multiplying and seeing whether the result is in fact the original expression.

$$(9x - 5)(9x + 5) \quad = 81x^2 + 45x - 45x - 25$$

$$= 81x^2 - 25$$

Example 2

Factor completely. $1 - 16y^2$

1) Check for a GCF. There is none.

2) Notice this is a "difference." Is each term a perfect square? Yes.

$$1 - 16y^2 = (1 - 4y)(1 + 4y)$$

3) Check work by multiplying and seeing whether the result is the original expression.

$$(1 - 4y)(1 + 4y) \quad = 1 - 4y + 4y + 16y^2$$

$$= 1 - 16y^2$$

Example 3

Factor completely. $2x^4 - 32$

1) Check for a GCF. The number "2" is a common factor; factor it out.

$$2x^4 - 32 \quad = 2(x^4 - 16)$$

2) Notice the expression in the bracket is a difference of squares. Factor it.

$$= 2(x^2 - 4)(x^2 + 4)$$

3) The first binomial is itself a difference of squares and so also must be broken down into its factors.

$$= 2(x - 2)(x + 2)(x^2 + 4)$$

Example 4

Factor completely. $20x^2 - 15$

1) Check for a GCF. The number "5" is a common factor.

$$20x^2 - 15 \quad = 5(4x^2 - 3)$$

2) Notice that the first term inside the bracket is a perfect square. Although the second term, the number "3," isn't a number we usually think of as a perfect square, it ***can*** be expressed as $\left(\sqrt{3}\right)\left(\sqrt{3}\right)$. This means we can factor the expression as a perfect square:

$$= 5\left(2x - \sqrt{3}\right)\left(2x + \sqrt{3}\right)$$

3) Factoring Trinomials

A trinomial is an algebraic expression consisting of 3 terms. Ones in the form $ax^2 + bx + c$ will often factor. If they do factor, they factor into two binomial factors.

Example 1

Factor completely. $x^2 + 7x + 12$

1) Check for a GCF. There is none.

2) Write down 2 empty brackets, each big enough for a binomial. In the "first" spot in each bracket, place the needed variable, so that when you multiply the first term in each binomial with each other, you will get the first term of the trinomial.

$$(x \qquad)(x \qquad)$$

3) Now, think of two numbers that
   - multiply to give the constant value (12)
   - add to give the value of the linear coefficient (7). ***(3 and 4 are the numbers)***

4) Write these numbers in the brackets. $(x + 3)(x + 4)$

5) Check by multiplying. $(x + 3)(x + 4) = x^2 + 3x + 4x + 12 = x^2 + 7x + 12$

Example 2

Factor completely. $3x^2 - 6x - 105$

1) Check for a GCF: $3x^2 - 6x - 105 = \quad 3(x^2 - 2x - 35)$

2) Write down 2 empty brackets, each big enough for a binomial. In the "first" spot in each bracket, place the needed variable, so that when you multiply the first term in each binomial with each other, you will get the first term of the trinomial.

$$3(x \qquad)(x \qquad)$$

3) Now, think of two numbers that
   - multiply to give the constant value (-35)
   - add to give the value of the linear coefficient (-2). ***(-7 and 5 are the numbers)***

4) Write these numbers in the brackets. $3(x - 7)(x + 5)$

5) Check by multiplying. $3(x - 7)(x + 5) = 3(x^2 + 5x - 7x - 35)$

$$= 3(x^2 - 2x - 35)$$

$$= 3x^2 - 6x - 105$$

Some trinomials have a leading coefficient that is not 1. They can be factored using several methods. Here are two examples, showing the "guess and check" method and the decomposition method.

Example 3

Factor completely, using "guess and check." $2x^2 - x - 21$

1) Check for a GCF. There is none.

2) Write down 2 empty brackets, each big enough for a binomial. In the "first" spot in each bracket, place the needed expression, so that when you multiply the first term in each binomial with each other, you get the first term of the trinomial. $(2x \quad)(x \quad)$

3) Make a list of the different possibilities for two numbers that multiply to give the constant value (-21)

-3, 7   3, -7   1, -21   -1, 21

4) Select one of the possibilities and write it in the brackets. Multiply to see if this choice gives the correct middle term of the original trinomial. Realize that putting the numbers in different locations creates a different trinomial.

$(2x-3)(x+7)$	=	$2x^2+14x-3x-21$	middle term is $11x$, not what we need
$(2x+7)(x-3)$	=	$2x^2-6x+7x-21$	middle term is x, not what we need
$(2x-7)(x+3)$	=	$2x^2+6x-7x-21$	middle term is $-x$, which ***is*** what we need

5) Final answer: $2x^2 - x - 21 = (2x-7)(x+3)$

This method works best when the leading coefficient and the constant term are prime numbers.

Keep track of what combinations you have tried so you don't accidentally keep trying the same ones over again!

Example 4

Factor completely, using decomposition, or the "AC" method. $3x^2 - 2x - 5$

1) identify A, B, and C : A = 3, B = -2, C = -5

2) calculate "AC" AC = (3)(-5) = -15

3) Find two numbers that multiply to give AC (-15) and add to give B (-2). ***-5 and 3***

4) Re-write the trinomial as a four-term expression. Split (or "decompose) the linear term into 2 terms whose coefficients are the numbers we just found in the previous step.

$$3x^2 - 2x - 5 \quad = 3x^2 \mathbf{- 5x + 3x} - 5$$

5) Factor out the GCF of the first two terms, then the GCF of the last two terms, as shown below:

$$3x^2 - 5x + 3x - 5 \quad = x(3x - 5) + 1(3x - 5)$$

6) Factor out the common binomial as a GCF: $(3x - 5)(x + 1)$

7) Check your answer by expanding.

$$(3x - 5)(x + 1) = 3x^2 + 3x - 5x - 5$$
$$= 3x^2 - 2x - 5$$

Part 3: Solving Equations

1) Linear Equations

"Linear" means that if the equation is graphed, you get a straight line. Variables in linear equations are not raised to any exponent other than 1.

Here are some steps to guide you when solving linear equations:

1) If there are brackets, distribute.

2) If there are fractions, eliminate them by multiplying each term by the least common denominator.

3) Collect all the terms containing the variable on one side of the equation and all terms that are constants on the other side of the equation.

4) Combine all like terms.

5) If the variable has a coefficient other than 1, divide to eliminate it.

Example 1

Solve for x.

$$\begin{aligned} 3(x-5) &= 21 \\ 3x - 15 &= 21 \\ 3x - 15 + 15 &= 21 + 15 \\ 3x &= 36 \\ x &= 12 \end{aligned}$$

Example 2

Solve for x.

$$\frac{3x}{8}+\frac{2}{5}=-2$$

$$40\left(\frac{3x}{8}+\frac{2}{5}\right)=40(-2)$$

$$\frac{40}{1}\left(\frac{3x}{8}\right)+\frac{40}{1}\left(\frac{2}{5}\right)=\frac{40}{1}\left(\frac{-2}{1}\right)$$

$$\frac{\overset{5}{\cancel{40}}}{1}\left(\frac{3x}{\underset{1}{\cancel{8}}}\right)+\frac{\overset{8}{\cancel{40}}}{1}\left(\frac{2}{\underset{1}{\cancel{5}}}\right)=\frac{40}{1}\left(\frac{-2}{1}\right)$$

$$5(3x)+8(2)=40(-2)$$

$$15x+16=-80$$

$$15x+16-16=-80-16$$

$$15x=-96$$

$$x=-\frac{96}{15}$$

2) Quadratic Equations

"Quadratic" means that if the equation is graphed, you get a parabola. The highest exponent to which a variable in a quadratic equation can be raised is the exponent 2.

To solve quadratic equations:

1) Do whatever algebra is required to get all terms on one side of the equation, and "0" on the other side of the equation.

2) Factor the expression you now see in your equation.
   a) Set each factor, one at a time, equal to "0."
   b) Solve each of these linear equations.

3) If the expression does NOT factor, use the quadratic formula to solve.

Example 1

Solve for *x*.

$$x^2 = 40 - 3x$$

$$x^2 + 3x = 40 - 3x + 3x$$

$$x^2 + 3x - 40 = 40 - 40$$

$$x^2 + 3x - 40 = 0$$

$$(x-5)(x+8) = 0$$

$$x - 5 = 0, \quad x = 5 \qquad x + 8 = 0, \quad x = -8$$

Solutions: $x = 5, \; x = -8$

Example 2

Solve for *x*. $$x^2+9x+5=0$$

Unfortunately, there are no numbers that multiply to 5 and also add to 9. This means we cannot factor this expression. However, we ***can*** solve it by using the quadratic formula.

> The quadratic formula is used to solve any quadratic equation in the form $ax^2+bx+c=0$. The solutions are found by substituting the values of a, b, and c into this formula:
>
> $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The equation is $x^2+9x+5=0$, so we substitute into the formula the values a = 1, b = 9, c = 5:

$$x=\frac{-9\pm\sqrt{(9)^2-(4)(1)(5)}}{2(1)}$$

This gives us two solutions: $x=\frac{-9+\sqrt{61}}{2}$, and $x=\frac{-9-\sqrt{61}}{2}$

Decimal approximations to the solutions are:

$x\approx-0.59$, and $x\approx-8.41$

Part 4: Radicals and Exponent Laws

1) Radicals

Here is a radical:

index

$$\sqrt{5}$$

radicand

The expression underneath the radical sign is called the ***radicand*** and the tiny number tucked in to the left of the radical sign is the ***index***. Often there is no number visible, in which case we know we are dealing with a square root – this means the index is equal to 2. Sometimes we need to simplify radicals, making the radicand as small as possible.

Example 1

Simplify the radical $\sqrt{32}$.

$$\begin{aligned}\sqrt{32} &= \sqrt{16\times 2}\\ &= \sqrt{16}\times\sqrt{2}\\ &= 4\sqrt{2}\end{aligned}$$

> Knowing a few of the smaller perfect squares and perfect cubes really helps make simplifying radicals easier:
>
> Perfect squares: 4, 9, 16, 25, 36, 49
>
> Perfect cubes: 8, 27, 64, 125

Example 2

Simplify the radical $\sqrt[3]{40}$.

$$\begin{aligned}\sqrt[3]{40} &= \sqrt[3]{8\times 5}\\ &= \sqrt[3]{8}\times\sqrt[3]{5}\\ &= 2\sqrt[3]{5}\end{aligned}$$

Example 1 (again)

Simplify the radical $\sqrt{32}$.

$$\begin{aligned}\sqrt{32} &= \sqrt{2\times 2\times 2\times 2\times 2}\\ &= 2\times 2\times\sqrt{2}\\ &= 4\sqrt{2}\end{aligned}$$

> Many students prefer to completely factor the radicand and look for repeated factors. This approach also works well.
>
> For a square-root, any PAIR of identical factors allows us to take its square-root out in front. For a cube-root, we need THREE identical factors to let us take the cube-root out in front.

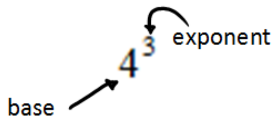
Example 2

Simplify the radical $\sqrt[3]{40}$.

$$\sqrt[3]{40} = \sqrt[3]{2\times 2\times 2\times 5} = 2\times\sqrt[3]{5} = 2\sqrt[3]{5}$$

2) Laws of Exponents

Here is a power:



Powers with positive exponents are a shorthand way of writing a ***multiplication*** of identical factors:

$$(4)^3 = (4)(4)(4) = 64$$

$$(2x)^5 = (2x)(2x)(2x)(2x)(2x) = 32x^5$$

$$5x^4 = 5(x)(x)(x)(x) = 5x^4$$

$$(-7)^2 = (-7)(-7) = 49$$

$$-7^2 = -\left[(7)(7)\right] = -49$$

Negative exponents tell us to ***divide*** by that number of factors, instead of multiplying:

$$(2)^{-3} = \frac{1}{(2)(2)(2)} = \frac{1}{8}$$

$$(4x)^{-2} = \frac{1}{(4x)(4x)} = \frac{1}{16x^2}$$

$$\begin{aligned}\frac{3}{x^{-4}} &= \frac{3}{\left(\frac{1}{x^4}\right)} \\ &= 3\times\left(\frac{x^4}{1}\right) \\ &= 3x^4\end{aligned}$$

Notice that when there is a negative exponent on an expression in the ***denominator*** of a fraction, it ends up "moving" to the top of the fraction, with a positive exponent.

If the exponent is equal to ***zero***, we get a very simple answer. Anything raised to the zero power is equal to 0 (except for 0^0, which is indeterminate).

$$\left(3xy^2\right)^0 = 1$$

$$\left(-7w\right)^0 = 1$$

Exponent Laws:

$$x^m x^n = x^{m+n}$$

$$(xy)^m = x^m y^m$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

$$\left(x^m\right)^n = x^{mn}$$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

Example 1

Simplify. Write answers with positive exponents only.

a) $\left(3m^4n^6\right)(2mn)^0(2m^2n)^3 = \left(3m^4n^6\right)(1)(8m^6n^3)$

$= 24m^{10}n^9$

b) $\dfrac{-28a^6b^{-2}c^6}{7a^{12}b^{-7}c^6} = \dfrac{-4b^{-2}b^7}{a^{-6}a^{12}}$

$= \dfrac{-4b^5}{a^6}$

c) $\dfrac{\left(-2ab^7\right)^3}{\left(-a^4b^2\right)^5} = \dfrac{(-8)\left(a^3\right)\left(b^{21}\right)}{(-1)^5\left(a^{20}\right)\left(b^{10}\right)}$

$= \dfrac{-8b^{11}}{-a^{17}}$

$= \dfrac{8b^{11}}{a^{17}}$

Here are some questions to try.

1. Evaluate each of the following. In your solution, show the step where you get the common denominator. Leave your answer as a fraction, not a decimal.

   a) $\frac{6}{11}+\frac{3}{5}$

   b) $-\frac{2\pi}{3}+\pi-\frac{3\pi}{4}$

   c) $\frac{3x}{5}+\frac{(x-4)}{2}$

2. Simplify each of the following completely.

   a) $\frac{6x+30}{20x}\times\frac{x+4}{(x+3)(x+5)}$

b) $\frac{15}{8} \div \frac{7}{12}$

3. Factor each of the following expressions completely.

a) $x^2 + x - 6$

b) $2x^2 + 11x + 12$

c) $5x^2 - 7x + 2$

d) $2x^2 + 7x + 6$

e) $6x^2 + 11x - 10$

f) $x^2 + 6x + 9$

g) $2x^2 - 13x + 6$

h) $x^2 - 6x + 8$

i) $4x^2 + 12x + 9$

j) $81x^2 - 144y^2$

k) $25 - 9x^2$

l) $3x^2 + 15x - 42$

m) $8x^2 + 4x - 60$

n) $48x^2 - 75$

o) $5x^2 - 35x - 90$

4. Solve for x. Clearly show the solution process.
   - If a solution is a fraction, leave it in reduced fractional form.
   (Some of the answers are kind of ugly!)
   - If you use the quadratic formula to get solutions, give the answers in two forms:
      - as exact answers
      - as approximate answers correct to 2 decimal places.

a) $5(6x - 1) + 4(3x + 8) = 2(3x - 7) + 11$

b) $\dfrac{2x-5}{3} + \dfrac{6x-1}{5} = \dfrac{5x+14}{15}$

c) $\frac{2}{5}x+\frac{5}{8}=8x-\frac{16}{3}$

d) $-x=2x^2-x$

e) $5x^2-x=3$

5. Simplify each radical.

a) $\sqrt{28x^4y}$

b) $\sqrt[3]{108w^2y^5}$

6. Simplify. Write answers with positive exponents only.

a) $(7ab)\left(-a^4b^3\right)^2\left(2a^5b^6\right)^{-1}$

b) $\dfrac{\left(3z^7\right)^2}{\left(3z^{-3}\right)^2}\times\dfrac{2x^4y^3}{2xy^{-6}}$

c) $\sqrt[3]{8x^6y^{12}}$

Additional Practice Questions

Fractions: Evaluate.

1. $1\frac{2}{3}+\frac{4}{5}=$

2. $-\frac{2}{3}-\frac{4}{5}=$

3. $-\frac{2}{5}\left(\frac{7}{2}-\frac{6}{4}\right)=$

4. $-3+\frac{10}{6}\times\frac{8}{12}=$

5. $-\frac{9}{4}\div1\frac{1}{2}=$

6. $\frac{mn}{nn}+\frac{n}{m}=$

7. $\frac{a}{mn}+\frac{b}{m^3}=$

Factoring:

Factor each of the following expressions completely.

8. $x^2 - 46x + 45 =$

9. $x^2 + 6x - 16 =$

10. $w^2 - 7wx - 44x^2 =$

11. $2x^2 + 12x + 10 =$

12. $5x^2 + 25x + 30 =$

13. $-10x^2 + 20x + 150 =$

14. $2x^2 + 3x + 1 =$

15. $3x^2 + 16x + 5 =$

16. $6x^2 + 17x + 5 =$

17. $10x^2 - 101x + 10 =$

18. $-4x^2 - 11x - 6 =$

19. $x^2 - 100 =$

20. $9x^2 - 16y^2 =$

21. $81x^2 - 144y^2 =$

Solving Linear Equations:

Solve for m in each of the equations. Clearly show the solution process.

22. $4(m-1) - 6m = -10(2m-1) - 1$

23. $2(m+1) + 4m = 4(m-2) + 6$

24. $\dfrac{5m+2}{2} = \dfrac{3m-1}{3}$

25. $\dfrac{m+5}{4} = \dfrac{2m+4}{5}$

26. $\dfrac{m}{4} + 5m = \dfrac{1}{2}m + 2$

27. $\dfrac{5}{2}(m-2) + 2 = 5$

Radicals and Exponent Laws

Simplify the following radicals.

28. $\sqrt{40}$

29. $\sqrt{27}$

30. $5\sqrt{12}+2\sqrt{75}$

31. $-3\sqrt{20}-\sqrt{45}$

32. $\sqrt{10}$ x $\sqrt{20}$

33. $\sqrt{24}$ x $\sqrt{3}$

34. $x=\dfrac{5\pm 10\sqrt{3}}{10}$

35. $x=\dfrac{5\pm\sqrt{25}}{5}$

Simplify and write without brackets.

36. $(3m^5)(4m^6)=$

37. $\dfrac{m^{50}m^{3}}{m^{40}}$ =

38. $\dfrac{m^2 m^8 (m^5)^2}{m^3 (m^2)^3} =$

39. $\dfrac{m^2 (m^8)^2 (m^5)^2}{m^3} =$

40. $\dfrac{4m^5 m^3 (m^3)^2}{6m^3 (m^2)^2} =$

Answers:

1. $\dfrac{37}{15}$ 2. $-\dfrac{22}{15}$ 3. $-\dfrac{4}{5}$ 4. $-\dfrac{17}{9}$ 5. $-\dfrac{3}{2}$ 6. $\dfrac{m^2+n^2}{nm}$ 7. $\dfrac{am^2+bn}{m^3 n}$

8. (x-45)(x-1) 9. (x+8)(x-2) 10. (w-11x)(w+4x) 11. 2(x+5)(x+1) 12. 5(x+3)(x+2)

13. -10(x-5)(x+3) 14. (2x+1)(x+1) 15. (3x+1)(x+5) 16. (2x+5)(3x+1) 17. (10x-1)(x-10)

18. –(4x+3)(x+2) 19.(x-10)(x+10) 20. (3x-4y)(3x+4y) 21. $9(3x-4y)(3x+4y)$

22. $\dfrac{13}{18}$ 23. -2 24. $-\dfrac{8}{9}$ 25. 3 26. $\dfrac{8}{19}$ 27. $\dfrac{16}{5}$ 28. $2\sqrt{10}$ 29. 3 $\sqrt{3}$

30. $20\sqrt{3}$ 31. $-9\sqrt{5}$ 32. $10\sqrt{2}$ 33. $6\sqrt{2}$ 34. $\dfrac{1\pm 2\sqrt{3}}{2}$ 35. 2 or 0

36. $12m^{11}$ 37. m^{13} 38. m^{11} 39. m^{25} 40. $\dfrac{2m^7}{3}$