More Factoring Practice WS

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See end of document for solutions!

Factoring Trinomials

The number one step in factoring is to check for a GCF. Once you have taken care of factoring out the GCF, you must decide if the remaining polynomial can be further factored. This section is about factoring trinomials (three-term polynomial). We will first look at trinomials with a leading coefficient of one and then trinomials with leading coefficients other than one.

Trinomials with a leading coefficient of one

<u>Example 1</u>: Factor x^2+5x+6

<u>Solution</u>: The nice thing about having a coefficient of one is you automatically know the first terms in the binomials. So we know the following: $(x\quad \_)(x\quad \_)$. We now need to fill in the last numbers along with their signs (+ or -). The last terms represent the factors of 6 whose sum is 5. There are only two sets of factors for 6: 1 & 6 or 2 & 3. The sum of 1 & 6 is 7, while the sum of 2 & 3 is 5. We now can finish the factoring: $(x+2)(x+3)$. (Note: it does not matter the order in which you write the factors. $(x+2)(x+3)=(x+3)(x+2)$).

You can check your answer by multiplying the two binomials – you should get the original problem. $(x+2)(x+3)=\underbrace{x^2}_{\text{F}}+\underbrace{3x+2x}_{\text{O+I}}+\underbrace{6}_{\text{L}}=x^2+5x+6$.

<u>Example 2</u>: Factor x^2+x-12

<u>Solution</u>: In this problem, we need to be careful of signs. Again, we already know the first terms of the binomials: $x^2+x-12=(x\quad \_)(x\quad \_)$. We need the factors of -12 (this tells us the factors must have opposing signs) whose sum is 1. The factors of 12 are: 1 &12, 2 & 6, and 3 & 4. In order for the sum to be 1, we must use the factors -3 and 4. The factorization is as follows: $x^2+x-12=(x-3)(x+4)$. Again, the order of the factors does not matter: $(x-3)(x+4)=(x+4)(x-3)$.

<u>Example 3</u>: Factor $x^2-4x+11$

<u>Solution</u>: We know the first terms of each binomial: $x^2-4x+11=(x\quad \_)(x\quad \_)$. We need the factors of 11 whose sum is -4. The only factors of 11 are 1 & 11 – it is not possible for the sum of 1 & 11 to equal -4; therefore, this trinomial can not be factored. We would call this trinomial "prime".

Factoring trinomials with leading coefficients other than one

<u>Example 4</u>: Factor $2x^2+7x+6$

<u>Solution</u>: We are no longer guaranteed the first terms to be *x*, so we need to be careful. The first step is to multiply the first coefficient and the constant: $(2)(6)=12$. We need to find the factors of 12 whose sum is the middle coefficient of 7. Again, the factors of 12 are: 1 &12, 2 & 6, and 3 & 4. Clearly, the sum of 3 & 4 is 7; therefore, we will re-write the middle term of the trinomial using the 3 and 4 (**order does not matter**):

$$2x^2+7x+6$$
$$2x^2+3x+4x+6$$

$$2x^2+3x+4x+6$$

Now we can factor by grouping: $x(2x+3)+2(2x+3)$

$$\boxed{(2x+3)(x+2)}$$

<u>Example 5</u>: Factor $4x^2-3x-7$

<u>Solution</u>: The first step is to multiply the first and last coefficients: $(4)(-7)=-28$. We need the factors of -28 whose sum is -3. The factors of 28 are: 1 & 28, 2 & 14, and 4 & 7. The factors needed are 4 & -7 (remember the sum must be -3). We now re-write the middle term of the trinomial and factor by grouping:

$$4x^2-3x-7$$
$$4x^2+4x-7x-7$$
$$4x(x+1)-7(x+1)$$
$$(x+1)(4x-7)$$

<u>Example 6</u>: Factor $4x^2-18x+8$

<u>Solution</u>: Remember to first step in factoring is to look for a GCF. In this case all terms have a 2 in common, so we will first factor out the 2: $2(2x^2-9x+4)$. Now we can try to factor the trinomial inside of the parentheses (the 2 does not go away). Multiply the first and last coefficients of the trinomial inside of the parentheses: $(2)(4)=8$. The factors of 8 whose sum is -9 are -1 & -8. We will rewrite the middle term and factor:

$$2\left[2x^2-8x-x+4\right]$$
$$2\left[2x(x-4)-1(x-4)\right]$$
$$2\left[(x-4)(2x-1)\right]$$
$$2(x-4)(2x-1)$$

Practice Problems

Factor completely:

1. $x^2 - 13x + 36$
2. $x^2 - 2x - 48$
3. $x^2 + 12x - 45$
4. $x^2 - 6x + 5$
5. $x^2 - 5x - 6$
6. $4x^2 + 24x - 64$
7. $2x^2 + 11x + 15$
8. $3x^2 - 13x + 14$
9. $5x^2 + 28x + 15$
10. $2x^2 - 3x - 35$
11. $2x^2 - 7x - 72$
12. $15x^2 - 33x - 36$
13. $5x^3 + 20x^2 - 60x$
14. $12x^4 + 60x^3 + 27x^2$

Solutions.
When you see "P" and "S" below, they are representing the words "Product" and "Sum"

Practice Problems

Factor completely:

1. $x^2-13x+36$

$(x-9)(x-4)$

2. $x^2-2x-48$

$(x+6)(x-8)$

3. $x^2+12x-45$

$(x-3)(x+15)$

4. x^2-6x+5

$(x-1)(x-5)$

5. x^2-5x-6

$(x+1)(x-6)$

6. $4x^2+24x-64$

$4(x^2+6x-16)$
$= 4(x-2)(x+8)$

7. $2x^2+11x+15$

P = 30
S = 11
5, 6

$= 2x^2+5x+6x+15$
$= x(2x+5)+3(2x+5)$
$= (2x+5)(x+3)$

8. $3x^2-13x+14$

P = 42
S = -13
-6, -7

$= 3x^2-6x-7x+14$
$= 3x(x-2)-7(x-2)$
$= (x-2)(3x-7)$

9. $5x^2+28x+15$

P = 75
S = 28
3, 25

$= 5x^2+3x+25x+15$
$= x(5x+3)+5(5x+3)$
$= (5x+3)(x+5)$

10. $2x^2-3x-35$

P = -70
S = -3
-10, 7

$= 2x^2-10x+7x-35$
$= 2x(x-5)+7(x-5)$
$= (x-5)(2x+7)$

11. $2x^2-7x-72$

P = -144
S = -7
-16, 9

$= 2x^2-16x+9x-72$
$= 2x(x-8)+9(x-8)$
$= (x-8)(2x+9)$

12. $15x^2-33x-36$

P = -60
S = -11
-15, 4

$= 3(5x^2-11x-12)$
$= 3(5x^2-15x+4x-12)$
$= 3[5x(x-3)+4(x-3)]$
$= 3(x-3)(5x+4)$

13. $5x^3+20x^2-60x$

$= 5x(x^2+4x-12)$
$= 5x(x-2)(x+6)$

14. $12x^4+60x^3+27x^2$

P = 36
S = 20
2, 18

$= 3x^2(4x^2+20x+9)$
$= 3x^2(4x^2+2x+18x+9)$
$= 3x^2[2x(2x+1)+9(2x+1)]$
$= 3x^2(2x+1)(2x+9)$