



PreCalc 11 Chapter 5 Assignment – hand in for completion marks

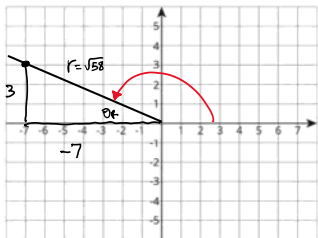
Name: Key

Complete the following questions showing all work and steps where applicable.

1. Each point below is a point on the terminal arm of a standard position angle θ . For each part of the question:

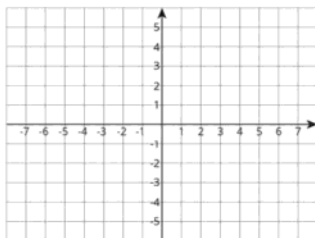
- Create a diagram showing the reference triangle with all side lengths labeled.
- Determine the primary trig ratios ($\sin \theta$, $\cos \theta$, $\tan \theta$), in simplified fractional form.
- Find the measure of angle θ , correct to the nearest degree.

a) point P(-7,3)



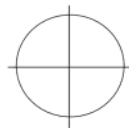
$$\begin{aligned} x^2 + y^2 &= r^2 & \sin \theta &= \frac{y}{r} = \frac{3}{\sqrt{58}} \\ (-7)^2 + (3)^2 &= r^2 & \theta_R &= \tan^{-1}\left(\frac{3}{-7}\right) \\ 49 + 9 &= r^2 & &= 23^\circ \\ 58 &= r^2 & \theta &= 180^\circ - 23^\circ \\ \sqrt{58} &= r & \theta &= 157^\circ \\ \tan \theta &= \frac{y}{x} = \frac{3}{-7} \end{aligned}$$

b) point P(6, -2)

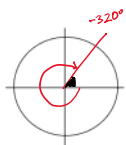


2. Sketch each angle in standard position. For each one, find the measure of its **reference angle**, the measure of one angle **coterminal** to the original angle, and **give the expression** that gives ALL angles coterminal to the original angle.

a) $\theta = 246^\circ$



b) $\theta = -320^\circ$



$$\begin{aligned} \theta_R &= 360^\circ - 320^\circ \\ \theta_R &= 40^\circ \end{aligned}$$

coterminal:

$$\begin{aligned} -320^\circ + 360^\circ &= 40^\circ \\ \text{OR} \quad -320^\circ - 360^\circ &= -680^\circ \end{aligned}$$

expression: $-320^\circ + 360^\circ n, n \in \mathbb{I}$

→ smallest positive coterminal angle

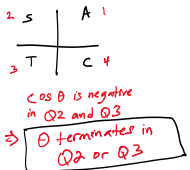
3. Find the measure of the **principal angle** for each angle listed below.

- a) -250°
- b) $-864^\circ - 864^\circ + 3(360^\circ) = \boxed{216^\circ}$
- c) 593°
- d) $1430^\circ - 3(360^\circ) = \boxed{350^\circ}$

4. For each equation below

- state the possible quadrants in which the angle θ can terminate
- for each quadrant, give the values for the other two trig ratios in exact fractional form
- find, correct to the nearest degree, the value of the reference angle, θ_r
- find, correct to the nearest degree, all values of θ that satisfy the equation for $0^\circ \leq \theta \leq 360^\circ$

a) $\cos \theta = \frac{4}{5}$
 $\cos \theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{4}{5}$



Q2

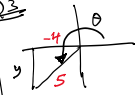


$(-4)^2 + y^2 = 5^2$
 $16 + y^2 = 25$
 $y^2 = 25 - 16$
 $y^2 = 9$
 $y = \pm 3$

In Q2, $y = +3$
 $\sin \theta = \frac{y}{r} = \frac{3}{5}$
 $\tan \theta = \frac{y}{x} = \frac{3}{-4}$

$\cos \theta = -\frac{4}{5}$
 $\theta_r = \cos^{-1}(\frac{4}{5})$
 $\theta_r \approx 37^\circ$
 $\theta = 180^\circ - \theta_r = \boxed{143^\circ}$

Q3



In Q3, $y = -3$
 $\sin \theta = -\frac{3}{5}$
 $\tan \theta = \frac{-3}{-4} = \frac{3}{4}$

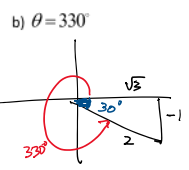
$\theta = 180^\circ + \theta_r$
 $= \boxed{217^\circ}$

b) $\tan \theta = -\frac{2}{5}$

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

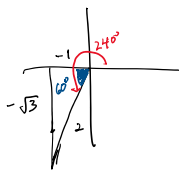
5. Use special triangle values to determine the primary trig ratios ($\sin \theta$, $\cos \theta$, $\tan \theta$) for each of the following angles. Give answers in EXACT form. (no decimal approximations!)

a) $\theta = 60^\circ$

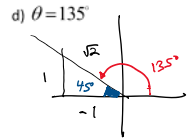


b) $\theta = 330^\circ$
 $\sin 330^\circ = -\frac{1}{2}$
 $\cos 330^\circ = \frac{\sqrt{3}}{2}$
 $\tan 330^\circ = -\frac{1}{\sqrt{3}}$

c) $\theta = 240^\circ$



$\sin 240^\circ = -\frac{\sqrt{3}}{2}$
 $\cos 240^\circ = -\frac{1}{2}$
 $\tan 240^\circ = \frac{-\sqrt{3}}{-1} = \sqrt{3}$



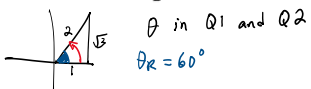
d) $\theta = 135^\circ$
 $\sin 135^\circ = \frac{1}{\sqrt{2}}$
 $\cos 135^\circ = -\frac{1}{\sqrt{2}}$
 $\tan 135^\circ = \frac{1}{-1} = -1$

6. Use special triangle values to find all values of θ that satisfy the equation for $0^\circ \leq \theta \leq 360^\circ$, for each equation below.

$\sqrt{3} \leftarrow y$

6. Use special triangle values to find all values of θ that satisfy the equation for $0^\circ \leq \theta < 360^\circ$, for each equation below.

a) $\sin \theta = \frac{\sqrt{3}}{2}$

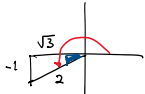


Q1: $\theta = 60^\circ$

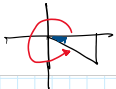
Q2: $\theta = 180^\circ - \theta_R = 120^\circ$

b) $\cos \theta = \frac{1}{\sqrt{2}}$

c) $\sin \theta = -\frac{1}{2}$

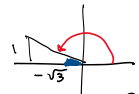


Q3: $\theta = 180^\circ + \theta_R = 210^\circ$

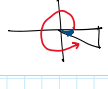


Q4: $\theta = 360^\circ - \theta_R = 330^\circ$

d) $\tan \theta = -\frac{1}{\sqrt{3}}$



Q2: $\theta = 180^\circ - \theta_R = 150^\circ$

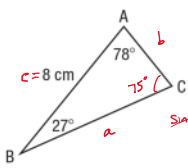


Q4: $\theta = 360^\circ - \theta_R = 330^\circ$

4

7. Solve each triangle, giving all values correct to 1 decimal place:

a)



$\angle C = 180^\circ - 78^\circ - 27^\circ$
 $\angle C = 75^\circ$

$\sin 78^\circ \cdot \left(\frac{a}{\sin 78^\circ}\right) = \left(\frac{8}{\sin 75^\circ}\right) \cdot \sin 78^\circ$

$a = \frac{8 \sin 78^\circ}{\sin 75^\circ}$

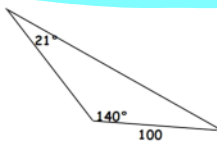
$a \doteq 8.1 \text{ cm}$

$\sin 27^\circ \cdot \left(\frac{b}{\sin 27^\circ}\right) = \left(\frac{8}{\sin 75^\circ}\right) \cdot \sin 27^\circ$

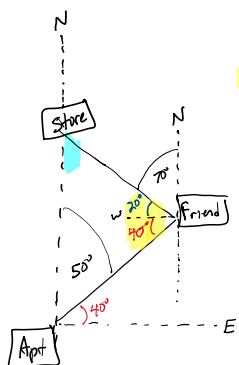
$b = \frac{8 \sin 27^\circ}{\sin 75^\circ}$

$b \doteq 3.8 \text{ cm}$

b)

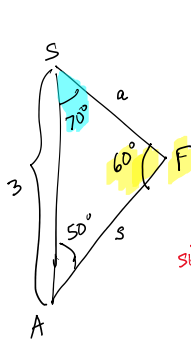


8. You walk to a store from your apartment. You don't walk directly there. Instead, you head $N50^\circ E$ to drop off something at your friend's place, then you change direction and travel $N70^\circ W$ to get to the store. The store is located exactly 3 km due north from your apartment. **In total, how far did you walk to get to the store?** Show your drawing with all angles and sides. Round your final answer, giving it correct to 1 decimal place.



$$\angle F = 20^\circ + 40^\circ = 60^\circ$$

$$\angle S = 180^\circ - 60^\circ - 50^\circ = 70^\circ$$



$$\frac{\sin 70^\circ}{s} = \left(\frac{3}{\sin 60^\circ} \right) \cdot \sin 70^\circ$$

$$s = \frac{3 \sin 70^\circ}{\sin 60^\circ}$$

$$s = 3.3 \text{ km}$$

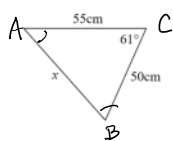
$$\frac{\sin 50^\circ}{a} = \left(\frac{3}{\sin 60^\circ} \right) \cdot \sin 50^\circ$$

$$a = \frac{3 \sin 50^\circ}{\sin 60^\circ}$$

$$a = 2.7 \text{ km}$$

$$\boxed{\text{Walked } 3.3 + 2.7 = 6 \text{ km}}$$

9. Solve the following triangle, giving answers correct to 1 decimal place.



$$x^2 = 55^2 + 50^2 - 2(55)(50)\cos 61^\circ$$

$$x^2 = 3025 + 2500 - 5500\cos 61^\circ$$

$$x = \sqrt{2858.547089} = 53.46538215$$

$$x \doteq 53.5 \text{ cm}$$

$$\frac{\sin B}{55} = \frac{\sin 61^\circ}{53.46538215}$$

Use full value, not rounded value

$$\sin B = \frac{55 \sin 61^\circ}{53.46538215}$$

$$\sin B = 0.8997239327$$

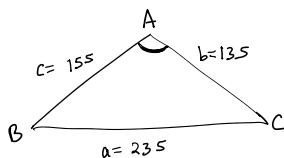
$$B = \sin^{-1}(0.899\dots)$$

$$B \doteq 64.1^\circ$$

$$A = 180^\circ - 61^\circ - 64.1^\circ$$

$$A \doteq 54.9^\circ$$

10. A triangular playground has sides of lengths 155 meters, 235 meters, and 135 meters. What are the measures of the angles between the sides, to the nearest tenth of a degree? Show your drawing with all angles and sides.



Find largest angle first.

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$A = \cos^{-1}\left(\frac{135^2 + 155^2 - 235^2}{2(135)(155)}\right)$$

$$A \doteq 108.1^\circ$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$B = \cos^{-1}\left(\frac{235^2 + 155^2 - 135^2}{2(235)(155)}\right)$$

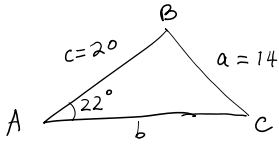
$$B \doteq 33.1^\circ$$

$$C = 180^\circ - 108.1^\circ - 33.1^\circ$$

$$C \doteq 38.8^\circ$$

11. Sketch the following triangles and solve, giving answers correct to 1 decimal place.
Hint: this is the ambiguous case.

a) For $\triangle ABC$ $\angle A = 22^\circ$, $a = 14$ cm, $c = 20$ cm.



$$\frac{\sin 22^\circ}{14} = \frac{\sin C}{20}$$

$$\sin C = \frac{20 \sin 22^\circ}{14}$$

$$C = \sin^{-1}\left(\frac{20 \sin 22^\circ}{14}\right)$$

$$\boxed{C = 32.4^\circ}$$

$$\Rightarrow B = 180^\circ - 22^\circ - 32.4^\circ$$

$$\boxed{B = 125.6^\circ}$$

$$\frac{b}{\sin 125.6^\circ} = \frac{14}{\sin 22^\circ}$$

$$b = \frac{14 \sin 125.6^\circ}{\sin 22^\circ} = \boxed{30.4 \text{ cm}}$$

OR

$$C = 180^\circ - 32.4^\circ$$

$$\boxed{C = 147.6^\circ}$$

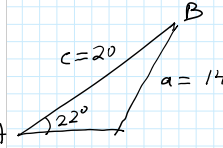
$$\Rightarrow B = 180^\circ - 22^\circ - 147.6^\circ$$

$$\boxed{B = 10.4^\circ}$$

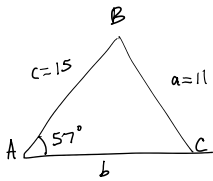
$$\frac{b}{\sin 10.4^\circ} = \frac{14}{\sin 22^\circ}$$

$$b = \frac{14 \sin 10.4^\circ}{\sin 22^\circ}$$

$$\boxed{b = 6.7 \text{ cm}}$$



b) For $\triangle ABC$ $\angle A = 57^\circ$, $a = 11$ cm, $c = 15$ cm.



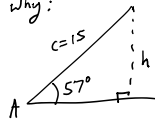
$$\frac{\sin C}{15} = \frac{\sin 57^\circ}{11}$$

$$\sin C = \frac{15 \sin 57^\circ}{11}$$

$$C = \sin^{-1}\left(\frac{15 \sin 57^\circ}{11}\right)$$

error! \Rightarrow $\boxed{\text{no triangle}}$

This is why:



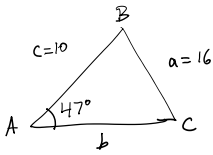
$$\sin 57^\circ = \frac{h}{15}$$

$$h = 15 \sin 57^\circ$$

$$h \approx 12.6$$

The shortest possible length for "a" is 12.6 cm,
 $a = 11$ cm is not long enough.

c) For $\triangle ABC$ $\angle A = 47^\circ$, $a = 16\text{cm}$, $c = 10\text{cm}$.



$$\frac{\sin C}{10} = \frac{\sin 47^\circ}{16}$$

$$\sin C = \frac{10 \sin 47^\circ}{16}$$

$$C = \sin^{-1}\left(\frac{10 \sin 47^\circ}{16}\right)$$

$$C \approx 27.2^\circ$$

$$\Rightarrow B = 180^\circ - 47^\circ - 27.2^\circ$$

$$B \approx 105.8^\circ$$

$$\frac{b}{\sin 105.8^\circ} = \frac{16}{\sin 47^\circ}$$

$$b = \frac{16 \sin 105.8^\circ}{\sin 47^\circ} \approx 21.1 \text{ cm}$$

OR

$$C = 180^\circ - 27.2^\circ$$

$$C = 152.8^\circ$$

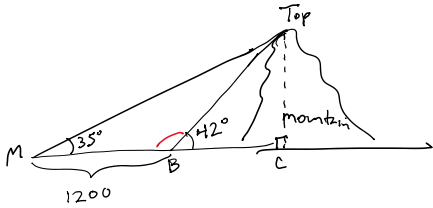
$$\Rightarrow B = 180^\circ - 152.8^\circ - 47^\circ$$

$$B = -19.8^\circ$$

Impossible!

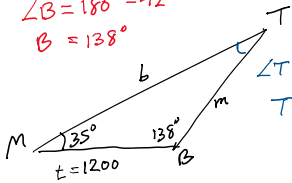
there is only one triangle.

12. Matt measures the angle of elevation from where he's standing to the peak of a mountain, to be 35° . Brooke is 1200 m closer to the mountain, along a level path. She measures the angle of elevation from her location to the peak of the mountain to be 42° . How high is the mountain, correct to 1 decimal place?



$$\angle B = 180^\circ - 42^\circ$$

$$B = 138^\circ$$



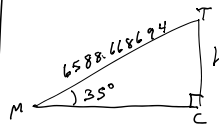
$$\angle T = 180^\circ - 138^\circ - 35^\circ$$

$$T = 7^\circ$$

$$\frac{b}{\sin 138^\circ} = \frac{1200}{\sin 7^\circ}$$

$$b = \frac{1200 \sin 138^\circ}{\sin 7^\circ}$$

$$b = 6588.668694$$



$$\sin 35^\circ = \frac{h}{6588.668694}$$

$$h = (\sin 35^\circ)(6588.668694)$$

$$h \approx 3779.1 \text{ m}$$