PC11 Ch 5 Hand-in Very Partial Key

Thursday, March 2, 2023 5:45 PM

PC11 Ch 5 Hand-in 2023

PreCalc 11 Chapter 5 Assignment – hand in for completion marks

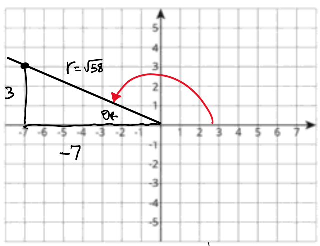
Name: Key

Complete the following questions showing all work and steps where applicable.

1. Each point below is a point on the terminal arm of a standard position angle θ. For each part of the question:

- Create a diagram showing the reference triangle with all side lengths labeled.
- Determine the primary trig ratios ($\sin\theta$, $\cos\theta$, $\tan\theta$), in simplified fractional form.
- Find the measure of angle θ, correct to the nearest degree.

a) point **P(-7,3)**



$x^2 + y^2 = r^2$
$(-7)^2 + (3)^2 = r^2$
$49 + 9 = r^2$
$58 = r^2$
$\sqrt{58} = r$

$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{58}}$

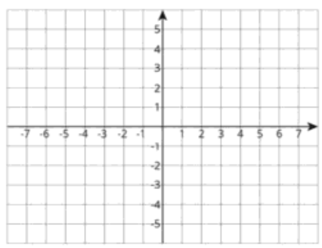
$\cos\theta = \frac{x}{r} = \frac{-7}{\sqrt{58}}$

$\tan\theta = \frac{y}{x} = \frac{3}{-7}$

$\theta_R = \tan^{-1}(+\frac{3}{7})$
$\doteq 23°$

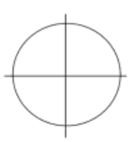
$\theta = 180° - 23°$
$\boxed{\theta = 157°}$

b) point **P(6, -2)**

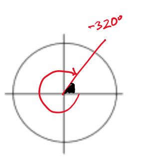


2. **Sketch** each angle in standard position. For each one, find the measure of its ***reference angle***, the measure of one angle ***coterminal*** to the original angle, and ***give the expression*** that gives ALL angles coterminal to the original angle.

a) $\theta = 246°$



b) $\theta = -320°$



$\theta_R = 360° - 320°$
$\boxed{\theta_R = 40°}$

coterminal:
$-320° + 360° = \boxed{40°}$
or
$-320° - 360° = \boxed{-680°}$

expression: $\boxed{-320° + 360°n,\ n \in I}$

smallest positive coterminal angle

3. Find the measure of the ***principal angle*** for each angle listed below.

a) -250°

b) -864°

$-864^\circ + 3(360^\circ)$

$= \boxed{216^\circ}$

c) 593°

d) 1430°

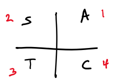
$1430^\circ - 3(360^\circ)$

$= \boxed{350^\circ}$

4. For each equation below

- state the possible quadrants in which the angle θ can terminate
- for each quadrant, give the values for the other two trig ratios in exact fractional form
- find, correct to the nearest degree, the value of the reference angle, θ_R
- find, correct to the nearest degree, all values of θ that satisfy the equation for $0^\circ \le \theta \le 360^\circ$

a) $\cos\theta = -\frac{4}{5}$

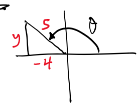


$\cos\theta = \frac{ADJ}{HYP} = \frac{-4}{5}$

cos θ is negative in Q2 and Q3

$\Rightarrow$ $\boxed{\theta \text{ terminates in Q2 or Q3}}$

Q2



$(-4)^2 + y^2 = 5^2$

$16 + y^2 = 25$

$y^2 = 25 - 16$

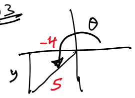
$y^2 = 9$

$y = \pm 3$

In Q2, $y = +3$

$\boxed{\sin\theta = \frac{y}{r} = \frac{3}{5} \quad \tan\theta = \frac{y}{x} = \frac{3}{-4}}$

Q3



In Q3, $y = -3$

$\boxed{\sin\theta = -\frac{3}{5} \quad \tan\theta = \frac{-3}{-4} = \frac{3}{4}}$

$\cos\theta = -\frac{4}{5}$

$\theta_R = \cos^{-1}(+\frac{4}{5})$

$\boxed{\theta_R \doteq 37^\circ}$

$\theta = 180^\circ - \theta_R = \boxed{143^\circ}$

$\theta = 180^\circ + \theta_R$

$= \boxed{217^\circ}$

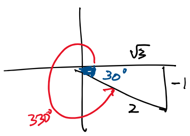
b) $\tan\theta = -\frac{2}{5}$

$\boxed{\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}}$

5. Use special triangle values to determine the primary trig ratios ($\sin\theta$, $\cos\theta$, $\tan\theta$) for each of the following angles. Give answers in EXACT form. (no decimal approximations!)

a) $\theta = 60^\circ$

b) $\theta = 330^\circ$

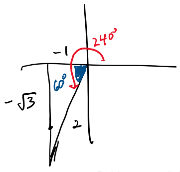


$\sin 330^\circ = -\frac{1}{2}$

$\cos 330^\circ = \frac{\sqrt{3}}{2}$

$\tan 330^\circ = -\frac{1}{\sqrt{3}}$

c) $\theta = 240^\circ$

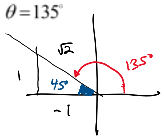


$\sin 240^\circ = -\frac{\sqrt{3}}{2}$

$\cos 240^\circ = -\frac{1}{2}$

$\tan 240^\circ = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

d) $\theta = 135^\circ$



$\sin 135^\circ = \frac{1}{\sqrt{2}}$

$\cos 135^\circ = -\frac{1}{\sqrt{2}}$

$\tan 135^\circ = \frac{1}{-1} = -1$

6. Use special triangle values to find all values of θ that satisfy the equation for $0^\circ \le \theta \le 360^\circ$, for each equation below.

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a) $\sin\theta = \frac{\sqrt{3}}{2}$ $\leftarrow y$, $\leftarrow r$

θ in Q1 and Q2

$\theta_R = 60^\circ$

$Q_1: \theta = \boxed{60^\circ}$

$Q_2: \theta = 180^\circ - \theta_R = \boxed{120^\circ}$

b) $\cos\theta = \frac{1}{\sqrt{2}}$

c) $\sin\theta = -\frac{1}{2}$ $\leftarrow y$, $\leftarrow r$

θ in Q3 and Q4

$\theta_R = 30^\circ$

$Q_3: \theta = 180^\circ + \theta_R = \boxed{210^\circ}$

$Q_4: \theta = 360^\circ - \theta_R = \boxed{330^\circ}$

d) $\tan\theta = -\frac{1}{\sqrt{3}}$ $\leftarrow y$, $\leftarrow x$

θ in Q2 and Q4

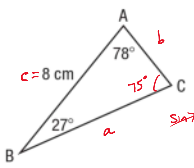
$\theta_R = 30^\circ$

$Q_2: \theta = 180^\circ - \theta_R = \boxed{150^\circ}$

$Q_4: \theta = 360^\circ - \theta_R = \boxed{330^\circ}$

7. Solve each triangle, giving all values correct to 1 decimal place:

a)



$\angle C = 180^\circ - 78^\circ - 27^\circ$

$\boxed{\angle C = 75^\circ}$

$\cancel{\sin 78^\circ} \cdot \left(\frac{a}{\cancel{\sin 78^\circ}}\right) = \left(\frac{8}{\sin 75^\circ}\right) \cdot \sin 78^\circ$

$a = \frac{8 \sin 78^\circ}{\sin 75^\circ}$

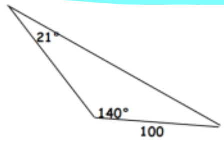
$\boxed{a \doteq 8.1 \text{ cm}}$

$\cancel{\sin 27^\circ} \cdot \left(\frac{b}{\cancel{\sin 27^\circ}}\right) = \left(\frac{8}{\sin 75^\circ}\right) \cdot \sin 27^\circ$

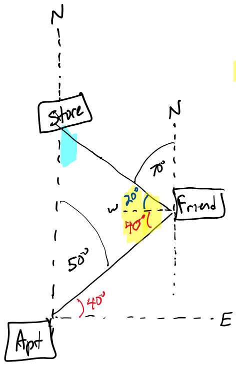
$b = \frac{8 \sin 27^\circ}{\sin 75^\circ}$

$\boxed{b \doteq 3.8 \text{ cm}}$

b)

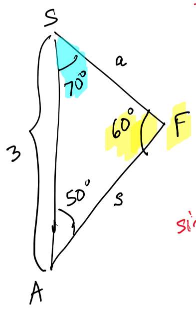


8. You walk to a store from your apartment. You don't walk directly there. Instead, you head $N50^\circ E$ to drop off something at your friend's place, then you change direction and travel $N70^\circ W$ to get to the store. The store is located exactly 3 km due north from your apartment. ***In total, how far did you walk to get to the store?*** *Show your drawing with all angles and sides.* Round your final answer, giving it correct to 1 decimal place.



$\angle F = 20^\circ + 40^\circ = 60^\circ$

$\angle S = 180^\circ - 60^\circ - 50^\circ$
$= 70^\circ$



$$\sin 70^\circ \cdot \left(\frac{s}{\cancel{\sin 70^\circ}}\right) = \left(\frac{3}{\sin 60^\circ}\right) \cdot \sin 70^\circ$$

$$s = \frac{3 \sin 70^\circ}{\sin 60^\circ}$$

$$s \doteq 3.3 \text{ km}$$

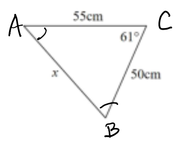
$$\sin 50^\circ \cdot \left(\frac{a}{\sin 50^\circ}\right) = \left(\frac{3}{\sin 60^\circ}\right) \cdot \sin 50^\circ$$

$$a = \frac{3 \sin 50^\circ}{\sin 60^\circ}$$

$$a \doteq 2.7 \text{ km}$$

$$\boxed{\text{Walked } 3.3 + 2.7 = 6 \text{ km}}$$

9. Solve the following triangle, giving answers correct to 1 decimal place.



$x^2 = 55^2 + 50^2 - 2(55)(50)\cos 61°$

$x^2 = 3025 + 2500 - 5500 \cos 61°$

$x = \sqrt{2858.547089} = 53.46538215$

$\boxed{x \doteq 53.5 \text{ cm}}$

$\frac{\sin B}{55} = \frac{\sin 61°}{53.46538215}$

Use full value, not rounded value

$\sin B = \frac{55 \sin 61°}{53.46538215}$

$\sin B = 0.8997239327$

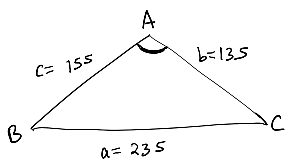
$B = \sin^{-1}(0.899\ldots)$

$\boxed{B \doteq 64.1°}$

$A = 180° - 61° - 64.1°$

$\boxed{A \doteq 54.9°}$

10. A triangular playground has sides of lengths 155 meters, 235 meters, and 135 meters. What are the measures of the angles between the sides, to the nearest tenth of a degree? *Show your drawing with all angles and sides.*



Find largest angle first.

$A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$

$A = \cos^{-1}\left(\frac{135^2+155^2-235^2}{2(135)(155)}\right)$

$\boxed{A \doteq 108.1°}$

$B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right)$

$B = \cos^{-1}\left(\frac{235^2+155^2-135^2}{2(235)(155)}\right)$

$\boxed{B \doteq 33.1°}$

$C = 180° - 108.1° - 33.1°$

$\boxed{C = 38.8°}$

11. Sketch the following triangles and solve, giving answers correct to 1 decimal place.
Hint: this is the ambiguous case.

a) For ΔABC $\angle A = 22^\circ$, $a = 14cm$, $c = 20cm$.

Two triangles here - solve them both.

(I will post more complete solutions later)

b) For ΔABC $\angle A = 57^\circ$, $a = 11cm$, $c = 15cm$.

No triangle possible - show how you know.

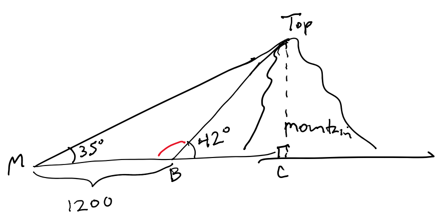
(I will post more complete solutions later)

c) For ΔABC $\angle A = 47^\circ$, $a = 16cm$, $c = 10cm$.

One triangle possible. Solve it.

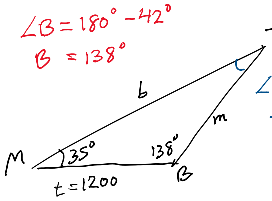
(I will post more complete solutions later)

12. Matt measures the angle of elevation from where he's standing to the peak of a mountain, to be 35°. Brooke is 1200 m closer to the mountain, along a level path. She measures the angle of elevation from her location to the peak of the mountain to be 42°. How high is the mountain, correct to 1 decimal place?



$\angle B = 180^\circ - 42^\circ$

$B = 138^\circ$



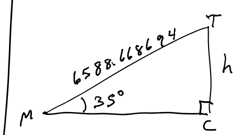
$\angle T = 180^\circ - 138^\circ - 35^\circ$

$T = 7^\circ$

$$\frac{b}{\sin 138^\circ} = \frac{1200}{\sin 7^\circ}$$

$$b = \frac{1200 \sin 138^\circ}{\sin 7^\circ}$$

$$b = 6588.668694$$



$$\sin 35^\circ = \frac{h}{6588.668694}$$

$$h = (\sin 35^\circ)(6588.668694)$$

$$\boxed{h \doteq 3779.1 \text{ m}}$$