What You Need to Know

Some of you may be wondering, "What am I supposed to know from before?" Here's what comes to mind, for the first unit. Look through the list. Which of these things do you need to review and/or learn?

Unit 1 – Functions and Transformations

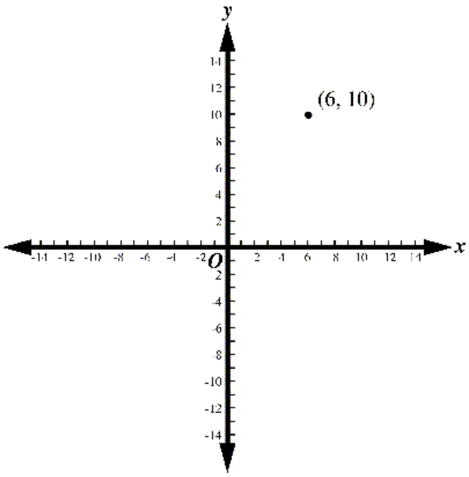
1. how to graph ordered pairs
2. how to graph by hand, making a table of values
3. how to graph on your graphing calculator, including finding the table of values
4. how to re-arrange an equation, to put it into the form "y =   "
5. how to substitute into f(x)
6. how to work with negative numbers
7. how to square a number, a monomial, a binomial
8. how to identify like terms
9. how to simplify an algebraic expression by adding/subtracting like terms
10. how to multiply algebraic terms together using laws of exponents
11. that dividing by zero is undefined
12. how to find the absolute value of a number
13. how to find the reciprocal of a number
14. how to factor: GCF, difference of squares, trinomials
15. how to solve a simple linear equation
16. how to solve a simple inequality
17. that "$\geq$" means "greater than or equal to"
18. that "$\leq$" means "lesser than or equal to"
19. that the equation of a vertical line is in the form "x = number"
20. that the equation of a horizontal line is in the form "y = number"
21. that when you square-root a number, you get two answers
22. that the maximum of a graph is the largest y-value it ever reaches
23. that the minimum of a graph is the smallest y-value it ever reaches

<u>vocabulary</u>: BEDMAS, binomial, coefficient, constant, denominator, difference, domain, exponent, FOIL, GCF, horizontal, linear, like terms, maximum, minimum, monomial, numerator, origin, parabola, perfect square, product, range, reciprocal, quadratic, slope, square-rooting, squaring, sum, trinomial, undefined, variable, vertex, vertical, x-intercept, y-intercept

Graphing

1) Plotting points

- The coordinate system, sometimes called the "Cartesian coordinate system," in honor of mathematician Rene Descartes, consists of an ***x-axis*** and ***y-axis***.



- Where these cross is the ***origin***, and it has coordinates (0, 0).
- We can plot any ordered pair (a, b) on the coordinate system.

the 1st number is the x- coordinate, and tells one how far left/right to go

the 2nd number is the y- coordinate, and tells one how far up/down to go

2) Graphing Lines

The equations of straight lines can be grouped into three types:

a) y = k, where k is a constant. These are horizontal, with slope of 0.
For example: $y = 3$, $y = 0$, $y = -5$

b) x = k, where k is a constant. These are vertical, with undefined slope.
For example: $x = 2.5$, $x = -4$, $x = 30$

c) Ones that contain both x and y terms, Ax + By = C.
For example: $3x – 4y = 7$, $-2x + 5y = 13$

When a linear equation is solved for y, we call this slope-intercept form, or

y = mx + b form

where "m" is the slope of the line and "b" is the y-intercept.

To graph easily, put the equation in **y = mx + b** form.

For example, $3x – 4y = 7$ becomes $-4y = -3x + 7$
$y = \frac{3}{4}x – 7/4$

So you know the slope is ¾ and the y-intercept is -7/4.

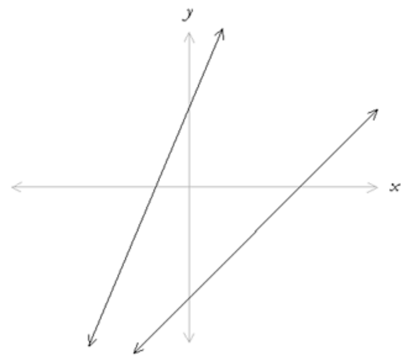
<u>slope</u> is a number that describes the steepness of a line. To calculate the slope of a line, use any two points on the graph and calculate as shown in the box below.

$$\textbf{Slope} = \frac{\text{difference in y-coordinates}}{\text{difference in x-coordinates}}$$

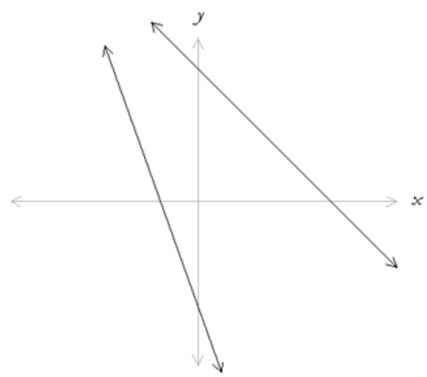
$$= \frac{y_1 - y_2}{x_1 - x_2} = \frac{rise}{run}$$

For example, the slope of the line that contains the points (2, 8) and (-3, 4) would be: $\frac{8-4}{2-(-3)} = \frac{4}{5}$

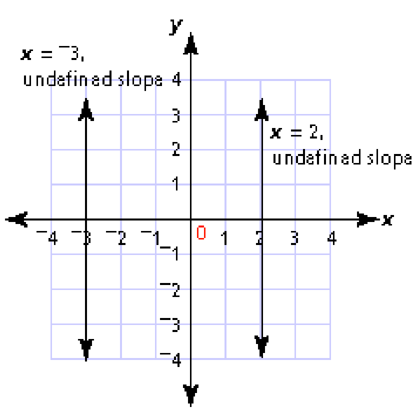
positive slope - lines with positive slope “lean uphill,” if you look at them from left to right.



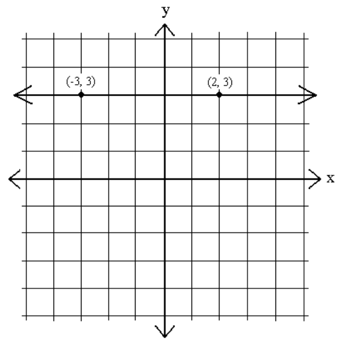
negative slope – lines with negative slope “lean downhill,” if you look at them from left to right.



undefined slope



zero slope



The **<u>y-intercept</u>** is the point where a line crosses the y-axis.

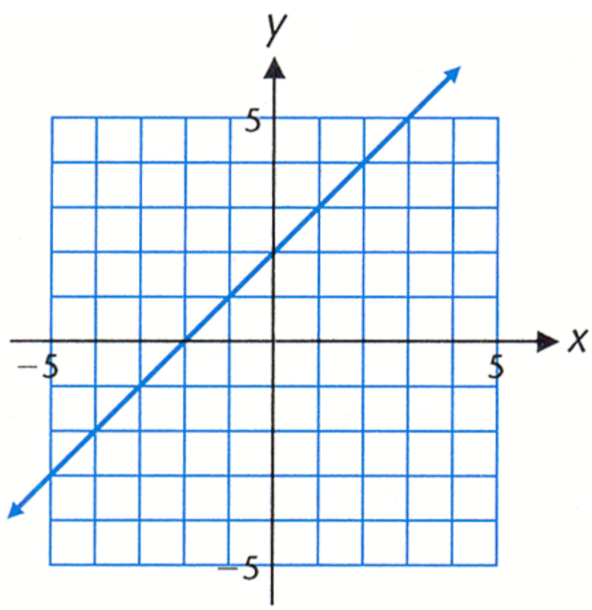
1) If the equation is put in **y = mx + b** form then the y-intercept is simply "b".
For example: $y = -2x + 5$ has a y-intercept at the point (0, 5).

2) Another way to find the y-intercept is to substitute the value "0 "into the equation for x, and solving. (This works because the y-intercept is always on the y-axis, and so its x-coordinate must be 0.)
For example:

$$3x - 4y = 12$$
$$3(0) - 4y = 12$$
$$0 - 4y = 12$$
$$-4y = 12$$
$$y = -3$$

The y-intercept is (0, -3).

3) **On the TI-83 graphing calculator**, you can find the y-intercept by first graphing the equation, and then TRACE, with a value of "0." This is the same thing as substituting the number zero into the equation. The calculator will tell you the y-value, which is the y-intercept.



<u>x-intercept</u>

1) To find the x-intercept of a line, the point where a line crosses the x-axis, simply substitute the value "0" into the equation for y, and solve.

For example:

$$2x + 6y = 18$$
$$2x + 6(0) = 18$$
$$2x = 18$$
$$x = 9$$

The x-intercept is (9, 0).

2) **On the TI-83 graphing calculator**, you can find the x-intercept by using the ZERO feature.

-Move the cursor (using the blue arrows) just to the left of the x-intercept for "left bound." Hit ENTER.

-Move the cursor just to the right of the x-intercept for "right bound." Hit ENTER.

-Move the cursor as close as you can get to x-intercept for "guess." Hit ENTER.

The calculator then gives the value of the x-intercept.

Simplifying Expressions

First some vocabulary:

constant – a number

variable – a letter used to represent a number

term – a constant, a variable, or a product of a constant and variables. Here are some examples:

$3x$ $-2x^3y^2$ 8 $5w$

coefficient – the number portion of a term, always written at the beginning of a term. For the terms listed above, their coefficients are:

$3x$	has a coefficient of "3"
$-2x^3y^2$	has a coefficient of "-2"
8	is a constant, doesn't have a coefficient
$5w$	has a coefficient of "5"

like terms – like terms have the same variables as each other, raised to exactly the same powers. The coefficients can be different.

$3x^2$ and $4y^2$ are NOT like
$3x^2$ and $2x$ are NOT like
$3x^2$ and $5x^2$ are like

An **algebraic expression** is what you get when you work with terms. You may need to add, subtract, multiply, or divide terms.

You can only add or subtract like terms.

Ex 1: $3x^2 + 4y^2$ cannot be simplified any further

Ex 2: $-5x^2 + 2x^2 – 7x + 12x – 3y + 2w$ can be simplified by combining the like terms with each other:

$-5x^2 + 2x^2 – 7x + 12x – 3y + 2w = -3x^2 + 5x – 3y + 2w$

Ex 3: $2x – 5 + 14 – 3x =$ $-x + 9$

You can multiply and divide terms even if they are not like terms.
Use the laws of exponents to multiply or divide terms.

Multiplication Law: $b^x b^y = b^{x+y}$	**Division Law:** $b^x \div b^y = b^{x-y}$
Power of a Power: $(b^x)^y = b^{xy}$	**Power of a Product:** $(ab)^x = a^x b^x$
Zero Exponent: $(b)^0 = 1$	

Some examples:

<u>Ex 1</u>: $4x^2y(8xy^3) = 32x^3y^4$

<u>Ex 2</u>: $3x(2x^2 - 8x + 14) = 6x^3 - 24x^2 + 42x$ *"Rainbow it in"*

<u>Ex 3</u>: $(2y)(3x)(5x^2) = (6xy)(5x^2) = 30x^3y$

<u>Ex 4</u>: $(x + 9)(x - 4) = x^2 - 4x + 9x - 36 = x^2 + 5x - 36$ *"FOIL"*

<u>Ex 5</u>: $4x(x - 5)(2x - 3)$
$= (4x^2 - 20x)(2x - 3)$
$= 8x^3 - 12x^2 - 40x^2 + 60x$
$= 8x^3 - 52x^2 + 60x$

<u>Ex 6</u>: $(3x^2y)^2$
$= (3x^2y)(3x^2y)$
$= 9x^4y^2$

<u>Ex 7</u>: $-(x^2 - 5x + 4) = -x^2 + 5x - 4$

<u>Ex 8</u>: $\dfrac{5x^3y^4}{20x^7y} = \dfrac{y^3}{4x^4}$

<u>Ex 9</u>: $(7x^2)^0 = 1$

<u>Ex 10</u>: $(x + 5)^2$
$= (x + 5)(x + 5)$
$= x^2 + 5x + 5x + 25$
$= x^2 + 10x + 25$

Solving Equations

To **solve** an equation means to find the numerical value that makes it true.

For example, when I solve the equation $3x = 6$ and I get $x = 2$ that means that I've found out the value of "x" that makes the first statement true..

1) Linear Equations

"Linear" means that if the equation is graphed, you get a straight line. Variables in linear equations are not raised to any power other than 1.

Here's one book's guide to on how to solve linear equations:

1) **If there are brackets, distribute.**
2) **If there are fractions, eliminate them by multiplying each term by the least common denominator.**
3) **If there are variables on the right hand side of the equation, use the inverse operation to move them to the left side.**
4) **Simplify each side.**
5) **If there are any constants on the left hand side of the equation, use the inverse operation to move them to the right side.**
6) **If there is a coefficient on the variable, use the inverse operation to eliminate it.**

Example 1: Solve for x.

$$\begin{aligned} 3(x-5) &= 21 \\ 3x - 15 &= 21 \\ 3x - 15 + 15 &= 21 + 15 \\ 3x &= 36 \\ x &= 12 \end{aligned}$$

Example 2: Solve for x.

$$\frac{3x}{8} + \frac{2}{5} = -2$$

$$(40)\frac{3x}{8} + (40)\frac{2}{5} = (40)(-2)$$

$$(5)(3x) + (8)(2) = -80$$

$$15x + 16 = -80$$

$$15x + 16 - 16 = -80 - 16$$

$$15x = -96, \qquad x = -\frac{96}{15}$$

2) Quadratic Equations

"Quadratic" means that if the equation is graphed, you get a parabola. The variable in a quadratic equation is raised to the power "2."

To solve quadratic equations:

1) **Do whatever algebra is required to get all terms on one side of the equation, and "0" on the other side.**
2) **Factor the expression you now have. This will give you two separate linear factors. (See next section to review factoring!)**
   a) **Set each factor, one at a time, equal to "0."**
   b) **Solve each of these linear equations.**
3) **If the expression does NOT factor, use the quadratic formula to solve.**

Example 1: Solve for x: $x^2 = 40 - 3x$

$x^2 + 3x = 40 - 3x + 3x$

$x^2 + 3x - 40 = 40 - 40$

$x^2 + 3x - 40 = 0$

$(x - 5)(x + 8) = 0$

$x - 5 = 0, x = 5$ $\quad$ $x + 8 = 0, x = -8$

Solutions: $x = 5, x = -8$

Example 2: Solve for x: $x^2 + 9x + 5 = 0$

Since this equation does not factor, we use the quadratic formula to solve it:

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting in the values a = 1, b = 9, c = 5, we find :

$x = \dfrac{-9 \pm \sqrt{(9)^2 - (4)(1)(5)}}{2(1)}$, which gives solutions: $x = \dfrac{-9 + \sqrt{61}}{2}$, $x = \dfrac{-9 - \sqrt{61}}{2}$

Factoring

To factor a number means to write it as a product of two or more numbers.
Example 1: 15 can be factored as 5 x 3. The three and the five here are called the factors of 15.

To factor completely just means that each factor must be as simple as possible – in other words, the factors themselves cannot be factored.
Example 2: 24 = 8 x 3, but this is not completely factored, because the 8 can itself be factored.
24 = 2 x 2 x 2 x 3 is factored completely

To factor an algebraic expression means to write it as a product of two or more algebraic factors.
Example 3: $3x^2 + 12xy$ can be factored as $3x(x + 4y)$

We'll look at 3 types of factoring.
1) **GCF**
2) **difference of squares**
3) **trinomials**

1) Factoring the Greatest Common Factor (GCF)

In all factoring questions, check for this kind of factoring first. See if there is a common factor that can "come out" of each of the terms in the algebraic expression. If there is, factor it out!

Example 4:
Factor completely: $9x^2y^4 + 18x^3y^2 + 15xy^2$

All three of the terms in this expression can be divided by $3xy^2$, so this is the greatest common factor that we will factor out in front:

$$9x^2y^4 + 18x^3y^2 + 15xy^2 = 3xy^2(3xy^2 + 6x + 5)$$

Example 5:
Factor completely: $10w^3 + 5w – 15w^2$

This time the greatest common factor (GCF) is 5w.

$$10w^3 + 5w – 15w^2 = 5w(2w^2 + 1 – 3w)$$

2) <u>Factoring the Difference of Squares</u>

difference – means to subtract
squares – here we mean perfect algebraic squares
Some numbers that are called **perfect squares** are 9, 16, and 64.
This is because they are numbers that result from squaring a number:

$(3)^2 = 9$ $\quad\quad$ $(8)^2 = 64$ $\quad\quad$ $(4)^2 = 16$.

Expressions like x^2, w^4, and $81x^2y^2$ are also called perfect squares because they result from multiplying something by itself:

$x^2 = (x)(x)$ $\quad\quad$ $w^4 = (w^2)(w^2)$ $\quad\quad$ $81x^2y^2 = (9xy)(9xy)$

A difference of squares factors easily:

$$\mathbf{a^2 - b^2 = (a - b)(a + b)}$$

<u>Example 6</u>:
Factor completely: $81x^2 - 25$

1) Check for a GCF. There is none.

2) Notice this is a "difference". Is each term a perfect square? Yes.

$$81x^2 - 25 = (9x - 5)(9x + 5)$$

3) Check by "FOIL"ing: Multiply to see if the answer is in fact the original expression.

<u>Example 7</u>: Factor completely: $2x^4 - 32$

1)Check for a GCF first. This time we <u>do</u> have one, the number 2:

$$2x^4 - 32 \quad = 2(x^4 - 16)$$

2) Notice the expression in the bracket is a difference of squares. Factor it:

$$= 2(x^2 - 4)(x^2 + 4)$$

3) Notice that the first binomial is itself a difference of squares.

$$= 2\mathbf{(x^2 - 4)}(x^2 + 4)$$

Factor this binomial.

$$= 2\mathbf{(x - 2)(x + 2)}(x^2 + 4)$$

2) <u>Factoring Trinomials</u>

A trinomial is an algebraic expression with 3 terms.

- **Ones that factor will generally have one squared term, one linear term, and one constant.**
- **They factor into two binomial factors**.

<u>Example 8</u>: Factor completely: $x^2 + 7x + 12$

1) Check for a GCF. There is none.

2) Write down 2 empty brackets, each big enough for a binomial.

3) In the "first" spot in each bracket, place the needed variable, so that when you multiply "first" x "first", you will get the first term.

$$(x \qquad)(x \qquad)$$

4) Now, think of two numbers that
   1) multiply to give the constant value (12),
   2) add to give the value of the linear coefficient (7).

5) Write these numbers in the brackets.

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

6) Check by "FOIL"ing.

<u>Example 9</u>: Factor completely: $x^2 - 4x - 21$

$$x^2 - 4x - 21 = (x - 7)(x + 3)$$

<u>Example 10</u>: Factor completely: $3x^2 - 6x - 105$

Check for a GCF first; this time there is one and it is 3.

$$\begin{aligned} 3x^2 - 6x - 105 &= 3(x^2 - 2x - 35) \\ &= 3(x - 7)(x + 5) \end{aligned}$$

Example11: Factor completely: $2x^2 - x - 21$

This trinomial has a leading coefficient of 2, not 1. But it can still be factored.

$2x^2 - x - 21 = (2x \quad)(x \quad)$

What numbers multiply to -21? -7, 3 3, -7 -21, 1 21, -1
None of these pairs add to "-1", but ***THAT DOESN'T MATTER!***

What makes this factoring possible is that the coefficient "2" in the first bracket will affect things.

Try something: $(2x + 7)(x - 3) = 2x^2 - 6x + 7x - 21 = 2x^2 + x - 21$
That wasn't it, but we were close.

Try changing the signs: $(2x - 7)(x + 3) = 2x^2 + 6x - 7x - 21 = \mathbf{2x^2 - x - 21}$
We got it!

This method is called **"guess and check"** and it works pretty well.
Keep track of what combinations you have tried so you don't accidentally do them over again!

decomposition" or "AC" method:

$2x^2 - x - 21$ is a trinomial, where A = 2, B = -1, and C = -21

1) calculate AC. AC = 2(-21) = -42

2) find two numbers that multiply to give AC, and add to give B.
   -7 and 6 work, since (-7)(6) = -42, and -7 + 6 = -1

3) separate the linear term into two terms, using as coefficients the values found in step 2
   $2x^2 - x - 21 = 2x^2 \mathbf{- 7}x + \mathbf{6}x - 21$

4) pull out the GCF of the first 2 terms, and then of the last two, as shown below:
   $2x^2 - 7x + 6x - 21 = x(2x - 7) + 3(2x - 7)$

5) rewrite the result in step 4 as the product of two binomials
   $x(2x - 7) + 3(2x - 7) = (x + 3)(2x - 7)$

6) check your work by multiplying out, using FOIL:
   $(x + 3)(2x - 7) = 2x^2 - x - 21$, so you know the factoring is correct